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APPLICATIONS OF MULTICONDUCTOR TRANSMISSION LINE THEORY
TO THE PREDICTION OF CABLE COUPLING

Computation of the Capacitance Matrices for Ribbon Cables

University of Kentucky

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ribbon cables is described in this report.

The per-unit-length charge distributions on the conductor and dielectric surfaces are described by Fourier series with the potential and displacement vector equations at these surfaces enforced as boundary conditions. A matrix inversion techniques is used to develop elements of the generalized and transmission line capacitance matrices.

Computer program GETCAP was written in a user-oriented format. The user simply describes the physical characteristics and GETCAP computes the per-unit-length capacitance matrices for the ribbon cable described.

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PREFACE

The Post-Doctoral Program at Rome Air Development Center is pursued via Project 9567 under the direction of Mr. Jacob Scherer. The Post-Doctoral Program is a cooperative venture between RADC and the participating universities: Syracuse University (Department of Electrical and Computer Engineering), the U.S. Air Force Academy (Department of Electrical Engineering), Cornell University (School of Electrical Engineering), Purdue University (School of Electrical Engineering), University of Kentucky (Department of Electrical Engineering), Georgia Institute of Technology (School of Electrical Engineering), Clarkson College of Technology (Department of Electrical Engineering), State University of New York at Buffalo (Department of Electrical Engineering), North Carolina State University (Department of Electrical Engineering), Florida Technological University (Department of Electrical Engineering), Florida Institute of Technology (College of Engineering), Air Force Institute of Technology (Department of Electrical Engineering), and the University of Adelaide (Department of Electrical Engineering) in South Australia. The Post-Doctoral Program provides, via contract, the opportunity for faculty and visiting faculty at the participating universities to spend a year full-time on exploratory development and operational problem-solving efforts with the post-doctoral splitting their time between RADC (or the ultimate customer) and the educational institutions.

The Post-Doctoral Program is totally customer-funded with current projects being undertaken for Air Defense Command (NORAD), Air Force Communications Service, Federal Aviation Administration, Defense Communications Agency, Aeronautical Systems Division (AFSC), Aero-Propulsion Laboratory (AFSC), and

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CHAPTER I

INTRODUCTION

Ribbon, or "flat-pack" cables, as described in this report, are a particular type of multiconductor cable consisting of n dielectric-coated conductors arranged in a linear array as shown in Fig. 1.1. Each conductor is cylindrical in shape, with all conductors in the cable of identical radii, r_c . Surrounding each conductor are identical, cylindrically-shaped homogeneous dielectrics, or insulations, of equal radii, r_d , and relative dielectric constant, ϵ_r . The conductors are uniformly separated by a distance, d , in the horizontal plane. In many types of ribbon cables, the conductor orientations are maintained in this linear array by bonding the adjacent dielectric insulations together. When the separation between conductors does not allow the dielectric surfaces to touch, a

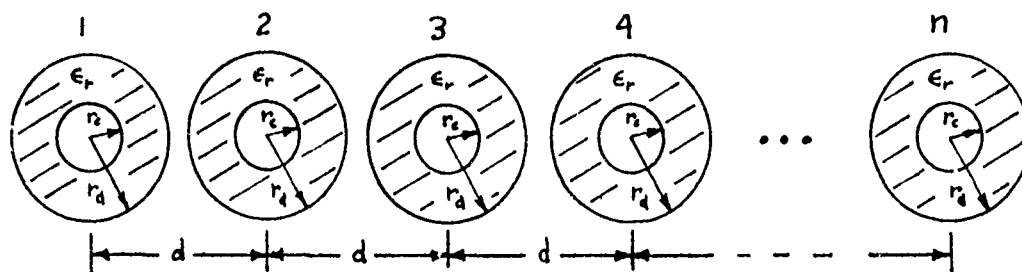


Figure 1.1 Orientation of ribbon or "flat-pack" cable.

thin film of the insulation connects the adjacent conductor insulations.

Ribbon cables have been used for many years in computer bus connections as in the Digital Equipment Corporation PDP-8 minicomputer [1]. They are recently finding greater usage in other types of systems such as aircraft and missiles. In these applications, the various conductors connect electronic devices at either end of the cable. Of considerable importance when using these cables is the ability to predict interference or crosstalk. Crosstalk is the unintentional coupling of signals from one wire onto another, resulting in possible bit errors in computer signals and the mixing of signals in analog systems.

Paul [2,13,19] has developed general techniques for characterizing a system of wires as a multiconductor transmission line, which can be used to predict crosstalk in ribbon cables. These techniques require that the per-unit-length transmission line capacitance and inductance matrices of the system, \underline{C} and \underline{L} , be obtained. The purpose of this report is to develop a numerical technique yielding these matrices for ribbon cables.

Approximations to the elements of the transmission line capacitance matrix can be obtained for cases with no dielectric insulation, providing the separation between the conductors is at least ten times the conductor radius. One such approximation will be derived in Chapter V, and will be used to develop an approximate expression for the trans-

mission line inductance matrix. Boast [3] describes an approximate method of determining the transmission line capacitance matrix for bare conductors above an infinite ground plane. The smallest ratio of conductor separation to wire radius must be greater than ten. In this case, one can assume that the per-unit-length charge on each conductor surface is uniformly distributed around the conductor periphery.

Ribbon cables, however, have a much smaller conductor separation than is required for these approximations to be valid, and in addition have dielectric insulations. Clements [4] showed that, in fact, the approximate formulas based on constant charge distributions are no longer sufficiently valid for close spacing and dielectric material surrounding the conductors.

Clements [4,9] has developed a general computer numerical technique for characterizing systems of circular conductors with circular dielectric insulations. This general technique is specialized in this report to the particular case of ribbon cables. As a result, many simplifications in the technique are made possible. This specialization of the general technique and the simplifications which result are described in detail in Chapter II. Certain special considerations in defining the boundary conditions and in choosing the matchpoints when applying the technique to ribbon cables are described in Chapter III. Chapter IV describes two alternate methods which were

investigated to reduce computer solution time; these methods did not yield valid results.

Chapter V describes the determination of the per-unit-length transmission line capacitance and inductance matrices, \underline{C} and \underline{L} , needed to analyze multiconductor systems. An approximate expression for the transmission line inductance matrix is also derived.

A computer program was written to implement the results of this report to provide a means of finding the transmission line capacitance and inductance matrices of ribbon cables. Chapter VI describes the operation of this computer program and the separate subroutines used. Results of the program are compared to both known results and to the approximate technique of Chapter V. Chapter VII is basically a users manual describing input formats and displaying typical outputs from the program. Finally, Chapter VIII summarizes the main points of the report, and Appendix B gives a full listing of the computer program.

CHAPTER II

DESCRIPTION OF THE METHOD

II.1 Capacitance and the Capacitance Matrix

To understand the concepts involved in determining the capacitance matrix for a system of wires, it is necessary to have a knowledge of the meaning of the term capacitance. This can be accomplished by considering the case of two arbitrary conducting bodies, as shown in Fig. 2.1. Q_1 and Q_2 are the total free charges on the bodies, and ϕ_1 and ϕ_2 are the potentials of the bodies with respect to some arbitrary reference point. The charges Q_1 and Q_2 can be related to the potentials of the bodies as shown in Eq. (2.1).

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \quad (2.1)$$



Figure 2.1. Two arbitrary conducting bodies.

The matrix

$$\underline{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (2.2)$$

is known as the "generalized" capacitance matrix [7].

Hereafter, $\underline{}$ will be used to denote matrix quantities, and an underbar, $\underline{}$, will denote nx1 column vectors. A matrix with m rows and n columns is of order mxn.

When the system of Fig. 2.1 is considered as an electrical network problem, it is customarily assumed that the bodies have been excited in such a way that charges of equal magnitude and opposite sign are present on the surfaces of the bodies. Enforcing the condition $Q=Q_1=-Q_2$ appropriate to network problems, Eq. (2.1) becomes

$$\begin{aligned} Q &= C_{11}\phi_1 + C_{12}\phi_2 \\ -Q &= C_{21}\phi_1 + C_{22}\phi_2 \end{aligned} \quad (2.3)$$

In most texts on electromagnetic theory, a network consisting of only two bodies is said to have a capacitance, C, between the bodies defined as [5,6,7]

$$C = \frac{Q}{V_o} \quad (2.4)$$

where V_o is the difference in potential of the bodies; i.e., $V_o = \phi_1 - \phi_2$.

Equation (2.3) can then be solved for the ratio $Q/(\phi_1 - \phi_2)$ yielding an expression for the capacitance

between the two bodies of Fig. 2.1 as [7]

$$C = \frac{Q}{\phi_1 - \phi_2} = \frac{C_{11}C_{22} - C_{12}C_{21}}{C_{11} + C_{22} + C_{12} + C_{21}} \quad (2.5)$$

The concept illustrated by Eq. (2.1) can be extended to the case of n charged bodies. Then, the $n \times n$ generalized capacitance matrix is defined by the equation

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \quad (2.6)$$

In matrix notation, Eq. (2.6) is written as

$$\underline{Q} = \underline{\tilde{C}} \underline{\phi} \quad (2.7)$$

where $\underline{\tilde{C}}$ is the generalized capacitance matrix of the system.

When considering infinitely long parallel conductors, total free charge is no longer finite. The charge on each conductor is customarily expressed as a function of its length. This per-unit-length free charge, q_f , can be related to the conductor potentials by the same relationship as in Eq. (2.6). This yields the matrix equation

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{nf} \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \cdots & \epsilon_{1n} \\ \epsilon_{21} & \epsilon_{22} & \cdots & \epsilon_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{n1} & \epsilon_{n2} & \cdots & \epsilon_{nn} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \quad (2.8a)$$

or

$$q_f = \epsilon \phi \quad (2.8b)$$

where q_{if} is the per-unit-length free charge on conductor i , and ϵ is the per-unit-length generalized capacitance matrix of the system.

II.2 Dielectric-Coated Conductors

Shown in Fig. 2.2 is a cross-section of two infinitely long circular conductors covered with a cylindrical dielectric. These dielectric-coated conductors will be referred to as wires. The dielectric material is assumed to be linear, homogeneous and isotropic. The quantities r_{c1} and r_{c2} in Fig. 2.2 designate the radii of the conductors, r_{d1} and r_{d2} designate the radii of the dielectrics, and ϵ_{r1} and ϵ_{r2} specify the relative dielectric constants of the die-

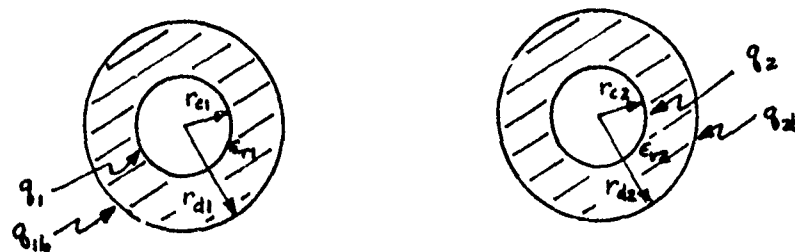


Fig. 2.2. Two dielectric-coated conductors.

electric materials relative to free space. The term "conductor surface" will designate the conductor-dielectric boundary, and the term "dielectric surface" will designate the dielectric-free space interface.

The potentials on each conductor cause quantities of "free" charge to build up on the surfaces of the conductors. These "free" charges produce an electric field in the region surrounding the conductors. The introduction of dielectric material into this field will cause the dipoles in the dielectric to separate their positive and negative charges to align themselves with the electric field. This separation results in a net drift of "bound" charge to the dielectric surface, q_{1b} and q_{2b} in Fig. 2.2. An equivalent amount of charge equal in magnitude but of opposite sign will be induced on the conductor surface. The total (bound plus free) charges q_1 and q_2 on the conductor surfaces are

$$q_1 = q_{1f} - q_{1b} \text{ and } q_2 = q_{2f} - q_{2b} \quad . \quad (2.9)$$

Thus, the "free" charge on the conductor is found by adding the values for the bound charge at the dielectric surface and the total charge at the conductor surface;

$$q_{1f} = q_1 + q_{1b} \text{ and } q_{2f} = q_2 + q_{2b} \quad . \quad (2.10)$$

Since the generalized capacitance matrix relates "free" charge to conductor potentials, the per-unit-length charges on both the conductor and dielectric surfaces must be found

in order to develop this matrix for dielectric-coated conductors.

II.3 Analytical Formulation of the Method

To find the generalized capacitance matrix of ribbon cables, a general computer numerical technique developed by Clements [4,9] is used, which utilizes the matrix methods described by Adams [7,8]. The technique is associated with the general "method of moments" [10] for solution of fields problems.

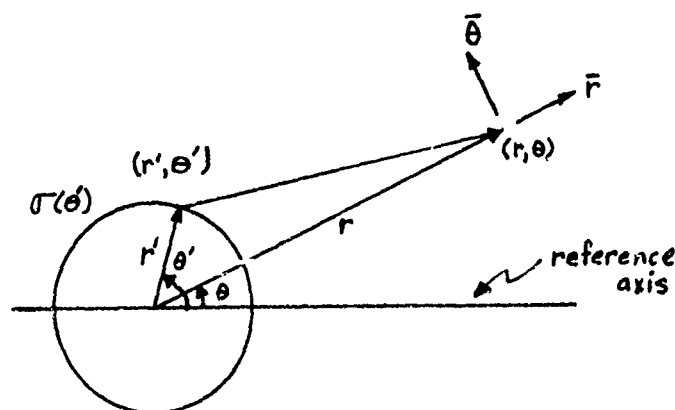


Figure 2.3. Fourier series charge distribution around a cylindrical surface.

In examining the method, the per-unit-length charge distribution, σ , around an infinitely long cylindrical surface as shown in Fig. 2.3, will be represented as a Fourier series,

$$\sigma(\theta') = \sigma_0 + \sum_{m=1}^k (\sigma_m \cos m\theta' + \hat{\sigma}_m \sin m\theta') \quad (2.11)$$

where k is the number of harmonics included in the representation. Then, the potential, with respect to an arbitrary

trary reference point, at a point (r, θ) outside this surface (i.e., $r > r'$) is [4] (ϵ_v is the permittivity of free space)

$$\begin{aligned} \phi(r, \theta) = & -\sigma_0 \frac{r' \ln r}{\epsilon_v} + \frac{1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \frac{(r')^{m+1} \cos m\theta}{mr^m} \\ & + \frac{1}{2\epsilon_v} \sum_{m=1}^k \hat{\sigma}_m \frac{(r')^{m+1} \sin m\theta}{mr^m} \end{aligned} \quad (2.12)$$

The reference potential term (defined in Appendix A) has been omitted in Eq. (2.12). It is shown in Appendix A that this reference term will vanish for a system with zero net charge and the reference point at infinity. Also, the electric field at the point (r, θ) can be found as [4]

$$\begin{aligned} \vec{E}(r, \theta) = & \frac{\sigma_0 \left(\frac{r'}{r}\right)}{\epsilon_v} \vec{r} + \frac{1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \left(\frac{r'}{r}\right)^{m+1} (\cos m\theta \vec{r} + \sin m\theta \vec{\theta}) \\ & + \frac{1}{2\epsilon_v} \sum_{m=1}^k \hat{\sigma}_m \left(\frac{r'}{r}\right)^{m+1} (\sin m\theta \vec{r} - \cos m\theta \vec{\theta}) \end{aligned} \quad (2.13)$$

where \vec{r} is a unit vector in the radial direction, and $\vec{\theta}$ is a unit vector in the tangential direction.

Similarly, if the point (r, θ) is inside the surface, that is, $r < r'$, [4]

$$\phi(r, \theta) = \frac{-\sigma_0 r' \ln r'}{\epsilon_v} + \frac{1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \frac{r^m \cos m\theta}{m(r')^{m-1}} + \frac{1}{2\epsilon_v} \sum_{m=1}^k \hat{\sigma}_m \frac{r^m \sin m\theta}{m(r')^{m-1}} \quad (2.14)$$

$$\begin{aligned} \bar{E}(r, \theta) = & \frac{-1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \left(\frac{r}{r^*} \right)^{m-1} (\cos m\theta \bar{r} - \sin m\theta \bar{\theta}) \\ & + \frac{-1}{2\epsilon_v} \sum_{m=1}^k \hat{\sigma}_m \left(\frac{r}{r^*} \right)^{m-1} (\sin m\theta \bar{r} + \cos m\theta \bar{\theta}) . \end{aligned} \quad (2.15)$$

Equations (2.12), (2.13), (2.14), and (2.15) can be used to find an approximation to the potential and electric field vector at any point due to a per-unit-length charge distribution on a cylindrical surface of the form of Eq. (2.11). A consideration of the geometry of the "flat-pack" cable results in a reduction of the number of terms in these equations.

Figure 2.4 shows the orientation of the wires in a "flat-pack" cable. All wires are identical and are oriented in a linear horizontal array.

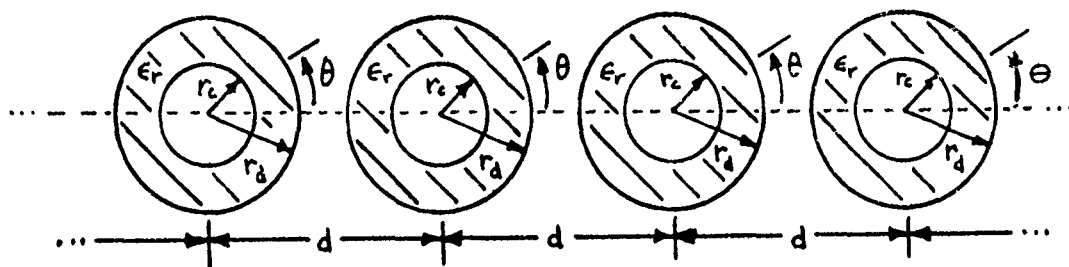


Figure 2.4 Physical geometry of "flat-pack" cable.

One would expect from the extreme symmetry of the "flat-pack" cable with respect to the line connecting centers of the wires, that the surface charges would be symmetric with respect to this line. This "even" function of the per-unit-length charge on the surfaces can be de-

scribed by constant and cosine terms only. In analytical form, the per-unit-length charge on a conductor surface is of the form

$$\sigma(\theta) = \sigma_0 + \sum_{m=1}^k \sigma_m \cos m\theta \quad (2.16)$$

Thus for "flat-pack" wire orientations, only $k+1$ of the full $2k+1$ Fourier coefficients need to be retained.

The potential and electric field equations for $r > r'$, Eq. (2.12) and Eq. (2.13), can then be reduced to

$$\phi(r, \theta) = -\sigma_0 \frac{r' \ln r}{\epsilon_v} + \frac{1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \frac{(r')^{m+1} \cos m\theta}{r^m} \quad (2.17)$$

and

$$\bar{E}(r, \theta) = \sigma_0 \left(\frac{r'}{r} \right) \bar{r} + \frac{1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \left(\frac{r'}{r} \right)^{m+1} (\cos m\theta \bar{r} + \sin m\theta \bar{\theta}) \quad (2.18)$$

Similarly, for $r < r'$, Eq. (2.14) and Eq. (2.15) become

$$\phi(r, \theta) = -\sigma_0 \frac{r' \ln r'}{\epsilon_v} + \frac{1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \frac{r^m \cos m\theta}{m (r')^{m-1}} \quad (2.19)$$

and

$$\bar{E}(r, \theta) = \frac{-1}{2\epsilon_v} \sum_{m=1}^k \sigma_m \left(\frac{r}{r'} \right)^{m-1} (\cos m\theta \bar{r} - \sin m\theta \bar{\theta}) \quad (2.20)$$

Similarly, the per-unit-length bound charge on a dielectric surface is of the form

$$\sigma'(\theta) = \sigma'_0 + \sum_{m=1}^1 \sigma'_m \cos m\theta \quad . \quad (2.21)$$

The potential and electric field vector due to the per-unit-length bound charge on the dielectric surface can be found from Eqns. (2.17) through (2.20) with σ replaced by σ' . The angle θ in the charge distributions in Eqns. (2.16) and (2.21) for each wire is with respect to the line connecting wire centers as shown in Fig. 2.4.

II.4 Matrix Organization in the Method

In the previous section, equations were developed for the potential and the electric field due to a charge distribution on a cylindrical surface, where the charge distribution is represented as a Fourier series. These equations are utilized in this section in a matrix formulation to solve for the per-unit-length generalized capacitance matrix.

Fourier series representations of $k+1$ terms are chosen for the charge distribution around each conductor surface, where k is the number of cosine terms in the series. Since the potential is equal at any point on the conductor surface, $k+1$ matchpoints on each conductor surface are used to define the potential resulting from the charge distributions on all the surfaces.

Similarly, Fourier series representations with $l+1$ terms are chosen to describe the bound charge distribution around the dielectric surface. The boundary conditions at

the dielectric surfaces require that the normal components of the displacement vector $\bar{D} = \epsilon \bar{E}$ must be equal just inside and just outside the dielectric boundary [7]. Therefore, 1+i matchpoints are chosen on each dielectric surface at which this condition will be enforced. The electric field will change by ρ_s/ϵ_v as a field point passes through the charged surface (where ρ_s is the surface charge in coulombs per square meter), but the normal component of the displacement vector will remain continuous across the boundary, i.e.,

$$\begin{aligned}\epsilon_i E_n^i - \epsilon_v E_n^o &= 0 \\ \epsilon_v (\epsilon_r E_n^i - E_n^o) &= 0 \\ \epsilon_r E_n^i - E_n^o &= 0\end{aligned}\tag{2.22}$$

where E_n^i and E_n^o are the normal components of the electric field just inside and just outside the boundary, respectively, ϵ_i and ϵ_r are the dielectric constant and the relative dielectric constant of the insulation, and ϵ_v is the permittivity of free space.

The matrix equation which results will be of the form

$$\underbrace{\begin{bmatrix} D_{11} & D_{11'} & D_{12} & D_{12'} & \dots & D_{1n} & D_{1n'} \\ D_{1'1} & D_{1'1'} & D_{1'2} & D_{1'2'} & \dots & D_{1'n} & D_{1'n'} \\ D_{21} & D_{21'} & D_{22} & D_{22'} & \dots & D_{2n} & D_{2n'} \\ D_{2'1} & D_{2'1'} & D_{2'2} & D_{2'2'} & \dots & D_{2'n} & D_{2'n'} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ D_{n1} & D_{n1'} & D_{n2} & D_{n2'} & \dots & D_{nn} & D_{nn'} \\ D_{n'1} & D_{n'1'} & D_{n'2} & D_{n'2'} & \dots & D_{n'n} & D_{n'n'} \end{bmatrix}}_{\tilde{D}} \begin{bmatrix} \underline{\sigma}_1 \\ \underline{\sigma}'_1 \\ \underline{\sigma}_2 \\ \underline{\sigma}'_2 \\ \vdots \\ \underline{\sigma}_n \\ \underline{\sigma}'_n \end{bmatrix} = \begin{bmatrix} \underline{\phi}_1 \\ 0 \\ \underline{\phi}_2 \\ 0 \\ \vdots \\ \underline{\phi}_n \\ 0 \end{bmatrix} \quad (2.23)$$

where unprimed quantities relate to the conductors, primed quantities relate to the dielectrics, and n equals the total number of wires in the cable. The $\underline{\sigma}_i$ terms are vectors of length $k+1$ containing the Fourier coefficients of free plus bound charge densities on the i -th conductor boundary, $\underline{\sigma}'_i$ are vectors of length $l+1$ containing the Fourier coefficients of the bound charge densities on the i -th dielectric boundary, and $\underline{\phi}_i$ are column vectors of length $k+1$, defining the conductor potential of the i -th conductor for $i=1, \dots, n$. These three vector quantities have an element organization as shown below:

$$\underline{\sigma}_i = \begin{bmatrix} \sigma_{i0} \\ \sigma_{i1} \\ \vdots \\ \sigma_{ik} \end{bmatrix} \quad \underline{\sigma}'_i = \begin{bmatrix} \sigma'_{i0} \\ \sigma'_{i1} \\ \vdots \\ \sigma'_{il} \end{bmatrix} \quad \underline{\phi}_i = \begin{bmatrix} \phi_i \\ \phi_i \\ \vdots \\ \phi_i \end{bmatrix} \quad (2.24)$$

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where the per-unit-length charge distribution on the conductor surface of the i -th wire is represented by

$$\sigma_i = \sigma_{i0} + \sum_{m=1}^k \sigma_{im} \cos m\theta \quad (2.25a)$$

and the per-unit-length bound charge distribution on the dielectric surface of the i -th wire is represented by

$$\sigma'_i = \sigma'_{i0} + \sum_{m=1}^l \sigma'_{im} \cos m\theta \quad (2.25b)$$

The submatrices of D have the significances noted below:

\tilde{D}_{ij} contains the contributions to the potential at the matchpoints of conductor i due to unit charge distribution coefficients on conductor j . This submatrix is of dimension $(k+1) \times (k+1)$.

$\tilde{D}_{ij'}$ contains the contributions to the potential at the matchpoints of conductor i due to unit charge distribution coefficients on dielectric j' . This submatrix is of dimension $(k+1) \times (l+1)$.

$\tilde{D}_{i'j}$ contains the contributions to the difference in the normal components of the displacement vector at the matchpoints on the surface of dielectric i' due to unit charge distribution coefficients on conductor j . This submatrix is of dimension $(l+1) \times (k+1)$.

$\tilde{D}_{i,j}$ contains the contributions to the difference in the normal components of the displacement vector at the matchpoints on the surface of dielectric i' due to unit charge distribution coefficients on dielectric j' . This submatrix is of dimension $(l+1) \times (l+1)$.

For convenience, the matchpoints on each conductor surface (dielectric surface) will be chosen according to the same rules for all conductor surfaces (dielectric surfaces). An example of this is shown in Fig. 2.5, where the matchpoints are denoted by X's.

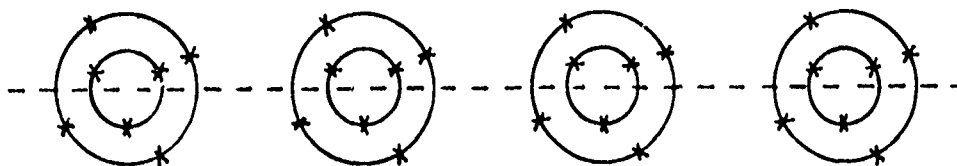


Figure 2.5. Matchpoint orientations for the conductor and dielectric surfaces.

Due to (1) the horizontal orientation of the flat-pack cable, (2) the identical size of all the wires (see Fig. 1.1), and (3) this particular method of choosing the matchpoints, many of the submatrices of \tilde{D} are identical. For example, \tilde{D}_{11} contains the contribution to the potential at the matchpoints of conductor 1 due to the components of its own charge distribution. Obviously, then,

$$\tilde{D}_{11} = \tilde{D}_{22} = \dots = \tilde{D}_{nn} \quad . \quad (2.26a)$$

Thus, the charge distribution on any conductor produces the same contribution to the potential at the matchpoints on its own surface. Also, the charge distribution on the surface of any dielectric produces the same contribution to the potential at the matchpoints on the conductor surface inside it:

$$\tilde{D}_{11'} = \tilde{D}_{22'} = \dots = \tilde{D}_{nn'} \quad . \quad (2.26b)$$

Similarly, it can be seen that

$$\begin{aligned} \tilde{D}_{1'1} &= \tilde{D}_{2'2} = \dots = \tilde{D}_{n'n} \\ \tilde{D}_{1'1'} &= \tilde{D}_{2'2'} = \dots = \tilde{D}_{n'n'} \quad . \end{aligned} \quad (2.26c)$$

This shows that all the submatrices on the main diagonal of \tilde{D} are identical, i.e.,

$$\begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{11'} \\ \tilde{D}_{1'1} & \tilde{D}_{1'1'} \end{bmatrix} = \begin{bmatrix} \tilde{D}_{22} & \tilde{D}_{22'} \\ \tilde{D}_{2'2} & \tilde{D}_{2'2'} \end{bmatrix} = \dots = \begin{bmatrix} \tilde{D}_{nn} & \tilde{D}_{nn'} \\ \tilde{D}_{n'n} & \tilde{D}_{n'n'} \end{bmatrix} \quad (2.27)$$

The same is true for the other diagonal partitions: hence the matrix in Eq. (2.23) can be written as

$$\begin{bmatrix} \underline{D}_0 & \underline{D}_{-1} & \underline{D}_{-2} & \cdots & \underline{D}_{-(n-1)} \\ \underline{D}_1 & \underline{D}_0 & \underline{D}_{-1} & \cdots & \underline{D}_{-(n-2)} \\ \underline{D}_2 & \underline{D}_1 & \underline{D}_0 & \cdots & \underline{D}_{-(n-3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{D}_{(n-1)} & \underline{D}_{(n-2)} & \underline{D}_{(n-3)} & \cdots & \underline{D}_0 \end{bmatrix} \begin{bmatrix} \underline{\sigma}_{-1}^* \\ \underline{\sigma}_{-2}^* \\ \underline{\sigma}_{-3}^* \\ \vdots \\ \underline{\sigma}_{-n}^* \end{bmatrix} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ \vdots \\ \underline{x}_n \end{bmatrix} \quad (2.28)$$

where

$$\underline{\sigma}_{-i}^* = \begin{bmatrix} \underline{\sigma}_{-i} \\ \underline{\sigma}'_{-i} \end{bmatrix}, \quad \underline{x}_i = \begin{bmatrix} \underline{\phi}_i \\ 0 \end{bmatrix},$$

and the subscripts on the \underline{D}_i submatrices in Eq. (2.28) are found from the subscripts of the corresponding submatrices of \underline{D} in Eq. (2.23) by subtracting the second subscript from the first. For example,

$$\begin{bmatrix} \underline{D}_{1n} & \underline{D}_{1n'} \\ \underline{D}_{1'n} & \underline{D}_{1'n'} \end{bmatrix} \equiv \underline{D}_{-(n-1)} \quad (2.29)$$

The matrix \underline{D} in Eq. (2.28) is in the block Toeplitz form allowing fast generation of the complete matrix, since only $2n-1$ of the original n^2 submatrices need be computed.

The matrix \underline{D} of equations (2.23) and (2.28) is inverted to obtain the matrix $\underline{T} = \underline{D}^{-1}$, and the equation becomes

$$\begin{bmatrix} \sigma_1 \\ \sigma_1' \\ \vdots \\ \sigma_2 \\ \sigma_2' \\ \vdots \\ \sigma_n \\ \sigma_n' \end{bmatrix} = \begin{bmatrix} T_{11} & T_{11}' & T_{12} & T_{12}' & \dots & T_{1n} & T_{1n}' \\ T_{1'1} & T_{1'1}' & T_{1'2} & T_{1'2}' & \dots & T_{1'n} & T_{1'n}' \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ T_{n1} & T_{n1}' & T_{n2} & T_{n2}' & \dots & T_{nn} & T_{nn}' \\ T_{n'1} & T_{n'1}' & T_{n'2} & T_{n'2}' & \dots & T_{n'n} & T_{n'n}' \end{bmatrix} \begin{bmatrix} \phi_1 \\ 0 \\ \vdots \\ \phi_n \\ 0 \end{bmatrix} \quad (2.30)$$

The total (bound plus free) per-unit-length charge, q_i , on the i -th conductor surface is the integral of the per-unit-length surface charge density, σ_i , over that surface. Thus,

$$q_i = \int_0^{2\pi} (\sigma_{i0} + \sum_{m=1}^k \sigma_{im} \cos m\theta) r_c d\theta \quad (2.31)$$

which reduces to

$$q_i = \int_0^{2\pi} \sigma_{i0} r_c d\theta = 2\pi r_c \sigma_{i0} \quad (2.32)$$

since the integration of a cosine function over a multiple of 2π is zero. The term r_c is the conductor radius, and σ_{i0} is the average value of the total per-unit-length charge density (the constant term of the Fourier expansion) on conductor i .

Similarly, the per-unit-length bound charge on the dielectric surface is

$$q_i' = 2\pi r_d \sigma_{i0}' \quad , \quad (2.33)$$

where r_d is the dielectric radius, and σ'_{i0} is the average value of the per-unit-length bound charge density on the i -th dielectric surface. As a consequence of Eq. (2.10), the per-unit-length free charge on the surface of conductor i is

$$q_{if} = q_i + q'_i \quad . \quad (2.34)$$

Thus, combining equations (2.32) and (2.33) yields the per-unit-length free charge on the i -th conductor,

$$q_{if} = 2\pi(r_c \sigma_{i0} + r_d \sigma'_{i0}) \quad . \quad (2.35)$$

From Eq. (2.30), the average per-unit-length charge density on the i -th conductor, σ_{i0} , (the first term in the $\underline{\sigma}_i$ vector) is

$$\sigma_{i0} = \underline{T}_{i1}^1 \underline{\phi}_1 + \underline{T}_{i2}^1 \underline{\phi}_2 + \cdots + \underline{T}_{in}^1 \underline{\phi}_n \quad (2.36)$$

where \underline{T}_{ij}^1 indicates a $1 \times n$ vector consisting of the first row of \underline{T}_{ij} . Similarly, the average per-unit-length charge density on the i -th dielectric, σ'_{i0} , is

$$\sigma'_{i0} = \underline{T}_{i,1}^1 \underline{\phi}_1 + \underline{T}_{i,2}^1 \underline{\phi}_2 + \cdots + \underline{T}_{i,n}^1 \underline{\phi}_n \quad . \quad (2.37)$$

Adding equations (2.36) and (2.37), according to equation (2.35), yields

$$\begin{aligned}
q_{if} &= 2\pi \left[r_c (\underline{T}_{i1}^1 \underline{\phi}_1 + \underline{T}_{i2}^1 \underline{\phi}_2 + \cdots + \underline{T}_{in}^1 \underline{\phi}_n) \right. \\
&\quad \left. + r_d (\underline{T}_{i,1}^1 \underline{\phi}_1 + \underline{T}_{i,2}^1 \underline{\phi}_2 + \cdots + \underline{T}_{i,n}^1 \underline{\phi}_n) \right] \\
&= 2\pi \sum_{j=1}^n \left[(r_c \underline{T}_{ij}^1 + r_d \underline{T}_{i,j}^1) \underline{\phi}_j \right] \quad (2.38)
\end{aligned}$$

Since $\underline{\phi}_i$ is a vector of equal conductor potentials, that is,

$$\underline{\phi}_j = \begin{bmatrix} \phi_i \\ \phi_i \\ \vdots \\ \phi_i \end{bmatrix}, \quad (2.39)$$

then Eq. (2.38) can be written as

$$q_{if} = 2\pi \sum_{j=1}^n \left\{ r_c \sum_{m=1}^n \underline{T}_{ij}^{1m} + r_d \sum_{m=1}^n \underline{T}_{i,j}^{1m} \right\} \phi_j \quad (2.40)$$

where \underline{T}_{ij}^{pq} is the element of \underline{T}_{ij} in the p-th row and q-th column. The desired generalized capacitance matrix is

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{nf} \end{bmatrix} = \begin{bmatrix} \underline{C}_{11} & \underline{C}_{12} & \cdots & \underline{C}_{1n} \\ \underline{C}_{21} & \underline{C}_{22} & \cdots & \underline{C}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{C}_{n1} & \underline{C}_{n2} & \cdots & \underline{C}_{nn} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \quad (2.41)$$

and the per-unit-length free charge on conductor i is

$$q_{if} = \sum_{j=1}^n (\underline{C}_{ij}) \phi_j \quad (2.42)$$

Thus comparing equations (2.40) and (2.42), the elements of the per-unit-length generalized capacitance matrix can be written as

$$C_{ij} = 2\pi \left\{ r_c \sum_{m=1}^n T_{ij}^{1m} + r_d \sum_{m=1}^n T_{i',j}^{1m} \right\} . \quad (2.43)$$

CHAPTER III

MATCHPOINT SELECTION

III.1 Introduction

In the previous chapter, a method for computing the generalized capacitance matrix of ribbon cables was developed. Several major considerations concerning choice of matchpoints will be discussed in this chapter. In the first part of this chapter, necessary criteria for choosing the position of the matchpoints are discussed. A matchpoint selection technique applicable to the ribbon cable problem was developed and is shown to meet these criteria. The last portion of the chapter provides some guidelines on the number of Fourier series coefficients and matchpoints required for the surface charges to yield accurate capacitance matrices.

III.2 Matchpoints and Nonsingularity of the \underline{D} Matrix

A key step in the method outlined in Chapter II is the inversion of the computed \underline{D} matrix. The necessary criterion to insure solution is that \underline{D} be nonsingular thus assuring that \underline{D}^{-1} exists. Therefore, the matchpoints should be chosen such that singular \underline{D} matrices are not generated.

A matrix is singular when any of its rows (or columns) can be expressed as a linear combination of the other rows

(columns). The most obvious case of this condition results when two rows (columns) are equal. Criteria for matchpoint selection which will guarantee that no two potential or displacement vector continuity equations are equal will be developed in this section. It should be pointed out, however, that these will constitute only necessary conditions for the nonsingularity of \underline{D} .

As described in Chapter II, the matrix equation defining \underline{D} is of the following form:

$$\begin{bmatrix} \underline{D}_{11} & \underline{D}_{11'} & | & \underline{D}_{12} & \underline{D}_{12'} & | & \cdots & | & \underline{D}_{1n} & \underline{D}_{1n'} \\ \underline{D}_{1'1} & \underline{D}_{1'1'} & | & \underline{D}_{1'2} & \underline{D}_{1'2'} & | & \cdots & | & \underline{D}_{1'n} & \underline{D}_{1'n'} \\ \underline{D}_{21} & \underline{D}_{21'} & | & \underline{D}_{22} & \underline{D}_{22'} & | & \cdots & | & \underline{D}_{2n} & \underline{D}_{2n'} \\ \underline{D}_{2'1} & \underline{D}_{2'1'} & | & \underline{D}_{2'2} & \underline{D}_{2'2'} & | & \cdots & | & \underline{D}_{2'n} & \underline{D}_{2'n'} \\ \vdots & \vdots & | & \vdots & \vdots & | & \ddots & | & \vdots & \vdots \\ \underline{D}_{n1} & \underline{D}_{n1'} & | & \underline{D}_{n2} & \underline{D}_{n2'} & | & \cdots & | & \underline{D}_{nn} & \underline{D}_{nn'} \\ \underline{D}_{n'1} & \underline{D}_{n'1'} & | & \underline{D}_{n'2} & \underline{D}_{n'2'} & | & \cdots & | & \underline{D}_{n'n} & \underline{D}_{n'n'} \end{bmatrix} \begin{bmatrix} \underline{\sigma}_1 \\ \underline{\sigma}_1' \\ \underline{\sigma}_2 \\ \underline{\sigma}_2' \\ \vdots \\ \underline{\sigma}_n' \end{bmatrix} = \begin{bmatrix} \underline{\phi}_1 \\ 0 \\ \underline{\phi}_2 \\ 0 \\ \vdots \\ \underline{\phi}_n \\ 0 \end{bmatrix} \quad (3.1)$$

It will be assumed that there are $k+1$ matchpoints around each of the conductor surfaces and $l+1$ matchpoints around each of the dielectric surfaces. The first block row of \underline{D}_{ij} and $\underline{D}_{i'j}$, submatrices in a partitioned row of Eq. (3.1) consists of $k+1$ equations for the potential at the conductor matchpoints of wire i . The next block row of \underline{D}_{ij} and $\underline{D}_{i'j}$, submatrices consist of $l+1$ equations containing

the differences in the normal components of the displacement vector at the matchpoints on the dielectric surface of wire i .

Examining the conditions for identical potential equations, the case where the identical equations are in different partitioned rows of the matrix in Eq. (3.1) will be considered first. The matchpoints corresponding to the two equations are located on different wires since the partitioning in Eq. (3.1) combines only those equations for a particular wire in each block row. Figure 3.1 illustrates two matchpoints, A and B, on the surfaces of the conductors of two different wires, i and j .

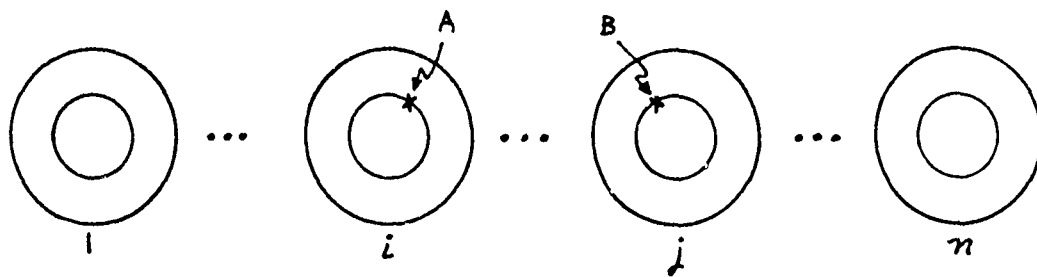


Figure 3.1 Matchpoints on two different wires in the cable.

Referring to the matrix equation (3.1), the rows associated with matchpoints A and B will be equal if the respective rows of the submatrices in each column partition are the same. That is,

$$D_{im}^A = D_{jm}^B \quad \text{for all } m=1, 1', \dots, n, n' \quad , \quad (3.2)$$

where D_{pq}^R indicates the row associated with matchpoint R in the submatrix D_{pq} . This implies that the contributions to the potentials at matchpoints A and B due to the components of the charge distributions on the m-th surface must be the same. It is obvious that this will be true only when matchpoints A and B are at the same location. For wires having dielectric insulations, however, matchpoints on two different conductors will never be coincident.

A similar analysis for the displacement vector continuity equations shows the same result which is illustrated in Fig. 3.2. For ribbon cables with separation greater than twice the dielectric radius, however, this will never occur, since the dielectric surfaces will not be touching.

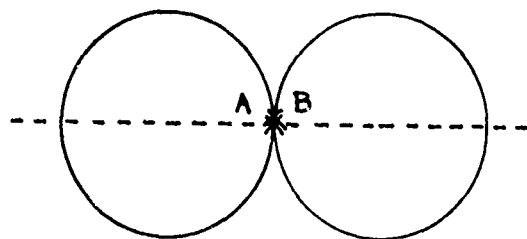
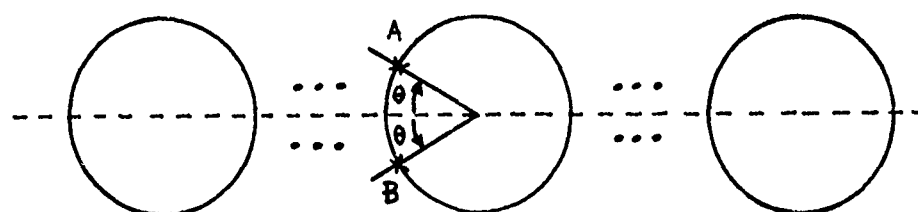


Figure 3.2 Coincident matchpoints from different surfaces.

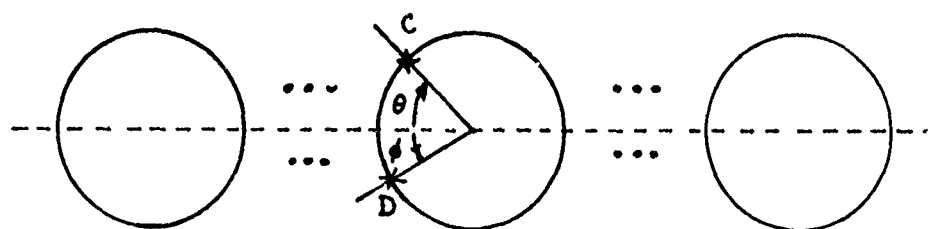
When the adjacent dielectric surfaces in a ribbon cable are touching, this may occur and a general matchpoint selection method should avoid this condition.

Another instance in which the D matrix in Eq. (3.1) will be singular occurs when two identical equations are in the same row partition, implying that the matchpoints associated with these identical equations are associated

with the same surface. As in the previous case, the elements of the two equations will be equal when the effects at the two matchpoints (on potential or continuity of the displacement vector) due to components of the charge distributions on the m -th surface are the same. This will result when choosing two matchpoints on the same surface of some wire as images of one another, above and below the line between the centers of the wires. Fig. 3.3(a) illustrates a typical occurrence of this condition.



(a) Matchpoints symmetrically disposed with respect to the center line on a charged surface (dielectric or conductor).



(b) Alternate choice of matchpoints on the surface.

Figure 3.3.

Matchpoints A and B in Fig. 3.3(a) are symmetrically disposed with respect to the charge distributions on all the

wires. The potentials (or fields) at these points due to terms of the Fourier series representation of the charge distributions (assumed to be an even function with respect to the line between centers) on any surface will be the same, thus generating two identical rows. This can be easily avoided by choosing the matchpoints on any surface according to the method in Fig. 3.3(b). If the angles θ and ϕ are not equal, the rows in the \underline{D} matrix associated with these matchpoints will not be equal.

A method for matchpoint selection must then meet two necessary criteria:

- 1) the matchpoints on two adjacent wires must not both be on the line between centers, and
- 2) the matchpoints on any surface must not be chosen symmetric with respect to the line between centers.

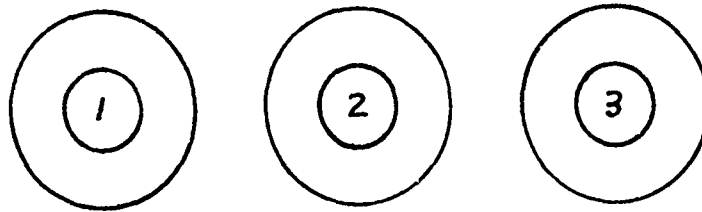
These criteria, of course, do not guarantee the nonsingularity of \underline{D} . They do, however, avoid the obvious cases in which a singular \underline{D} matrix will be generated.

III.3 Matchpoints and Symmetry of the \underline{C} Matrix

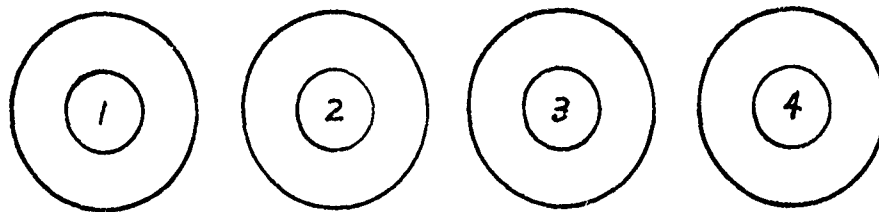
Due to the extreme symmetry of the "flat-pack" problem (conductor radii identical, dielectric radii identical, separation of the wires identical, and the wires oriented in a horizontal plane), some of the elements of \underline{C} must be identical. As an example, consider the case of three wires

shown in Fig. 3.4(a). The generalized capacitance matrix becomes

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{21} \\ C_{13} & C_{12} & C_{11} \end{bmatrix} \quad (3.3)$$



(a) A three-wire cable .



(b) A four-wire cable .

Figure 3.4 Three and four-wire cables illustrating the geometrical symmetry.

The resulting structure of \tilde{C} in Eq. (3.3) can be easily shown by recalling that C_{ij} is the ratio of the total per-unit-length free charge on the i -th conductor with a unit potential applied to the j -th conductor and all other conductor potentials set equal to zero; i.e.,

$$C_{ij} = \frac{q_{if}}{\phi_j} \bigg|_{\substack{\phi_m=0 \\ m \neq j}} \quad (3.4)$$

Clearly then from geometrical considerations

$$\begin{aligned} C_{33} &= C_{11} \\ C_{23} &= C_{21} \\ C_{32} &= C_{12} \\ C_{31} &= C_{13} \end{aligned} \quad (3.5)$$

as indicated in Eq. (3.3).

For four wires \tilde{C} becomes

$$\tilde{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{24} & C_{23} & C_{22} & C_{21} \\ C_{14} & C_{13} & C_{12} & C_{11} \end{bmatrix} \quad (3.6)$$

Thus it can be shown from Eq. (3.4) that for this four wire case (see Fig. 3.4(b)),

$$\begin{aligned} C_{33} &= C_{22} \\ C_{44} &= C_{11} \\ C_{31} &= C_{24} \\ C_{32} &= C_{23} \\ C_{41} &= C_{14} \\ C_{42} &= C_{13} \\ C_{43} &= C_{12} \end{aligned} \quad (3.7)$$

Therefore, the matchpoint selection technique should yield this "geometrical symmetry". Matrices with these properties are called centro-symmetric [18]. An nxn centro-symmetric matrix M has the property that $\tilde{M} = \tilde{S}MS$ where \tilde{S} is an nxn matrix with ones on the cross-diagonal and zeros elsewhere; i.e., $S_{ij} = 1$ for $i+j = n+1$ and zeros elsewhere [18].

Reciprocity relations [11] also show that the generalized capacitance matrix is symmetric with respect to the main diagonal. Thus, the generalized capacitance matrices for three and four wire cables can be written as

$$\tilde{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{12} \\ C_{13} & C_{12} & C_{11} \end{bmatrix} \quad (3.8a)$$

$$\tilde{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & C_{22} & C_{23} & C_{13} \\ C_{13} & C_{23} & C_{22} & C_{12} \\ C_{14} & C_{13} & C_{12} & C_{11} \end{bmatrix} \quad (3.8b)$$

Therefore, the generalized capacitance matrix should be symmetric with respect to both the main diagonal and the cross-diagonal and those elements above the dashed lines in Eq. (3.8) completely characterize the matrix.

Another necessary criterion, then, in choosing the matchpoints for the method described in Chapter II is that the computed capacitance matrix be symmetric with respect to

both the main diagonal and the cross-diagonal.

III.4 A Method for Matchpoint Selection

In the implementation of the general technique, many choices of the number of matchpoints and expansion terms for each surface are possible. However, one particular choice will be used in this report and the resulting computer program. The number of terms in the Fourier series distributions associated with each dielectric surface of each wire will be the same and will be denoted by NFD. Also, the number of expansion terms associated with each conductor surface of each wire will be the same and will be denoted by NFC. The number of matchpoints associated with each dielectric surface of each wire will be equal to the number of expansion coefficients associated with this surface; i.e., $NFD = l+1$. Similarly, the number of matchpoints associated with each wire will be equal to the number of expansion coefficients associated with this surface; i.e., $NFC = k+1$.

During the course of this work, a recursive method for selection of the matchpoint positions was developed. This will be described and then shown to meet the necessary criteria. The method is dependent on the number of Fourier coefficients chosen for the distribution of charge around the surface. The relation assumes the matchpoints are evenly divided around the surface, so that the only independent variable is the rotation of the matchpoint set from the zero degree axis. This is illustrated in Fig. 3.5 for five

matchpoints on a particular surface. The angle θ used in the potential and electric field expressions in Chapter II will be chosen with respect to the horizontal line between centers, as shown in Fig. 2.4 and Fig. 3.5.

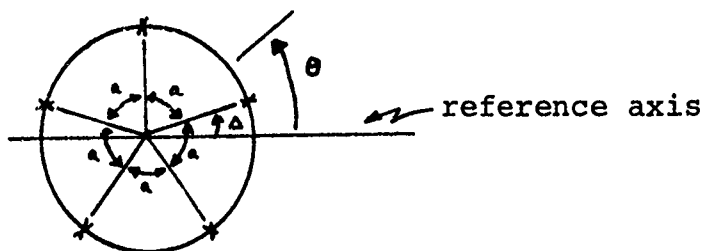


Figure 3.5 Example of matchpoint selection.

The angle, a , between any two adjacent matchpoints is determined by

$$a = \frac{2\pi}{NFS} \quad , \quad (3.9)$$

where NFS indicates the number of Fourier coefficients (matchpoints) associated with the surface (conductor or dielectric). For this particular case of five matchpoints,

$$a = \frac{2\pi}{5} = 72^\circ \quad .$$

The angle Δ , the rotation of the matchpoints from the $\theta=0$ reference, is given by

$$\Delta = \frac{\pi}{(2 \times NFS)} \quad . \quad (3.10)$$

Thus for the case of NFS=5, as above,

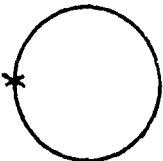
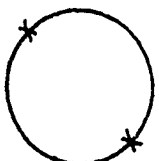
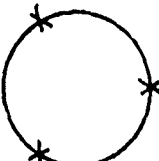
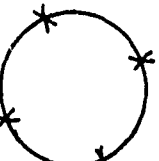
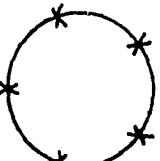
$$\Delta = \frac{\pi}{(2 \times 5)} = \frac{\pi}{10} \quad .$$

The matchpoints then lie at angles $\frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$ from the reference.

Table 1 shows the typical matchpoint selections using the formulas in Eq. (3.9) and Eq. (3.10) for cases where the number of matchpoints is in the range of 1 to 10.

For all the cases shown in Table 1, the matchpoints on any particular surface (conductor or dielectric) are not symmetric with respect to the line connecting wire centers, nor do the matchpoints occur on the line connecting wire centers. This is true for all finite values of NFS. In general, then, this matchpoint selection technique will always meet the criteria which were necessary in avoiding \tilde{D} matrices in which two rows of the potential equations or the displacement vector continuity equations are identical.

It was shown in Section III.3 that the generalized capacitance matrix should be symmetric about the main and cross-diagonals. This would be true if the exact charge distributions on the conductor and dielectric surfaces are obtained. In applying this technique, an approximation to the true charge distributions will be obtained. In the limit as the number of matchpoints on each surface increases without bound, these approximate charge distributions will approach the true charge distributions. In order to minimize the computation time and required core storage, how-

NFS	a	Δ	Pictorial
1	2π	$\frac{\pi}{2}$	
2	π	$\frac{\pi}{4}$	
3	$\frac{2\pi}{3}$	$\frac{\pi}{6}$	
4	$\frac{\pi}{2}$	$\frac{\pi}{8}$	
5	$\frac{2\pi}{5}$	$\frac{\pi}{10}$	

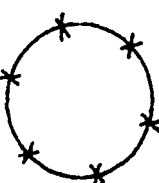
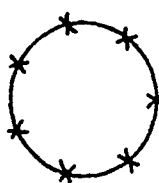
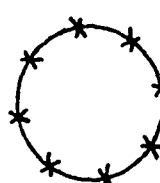
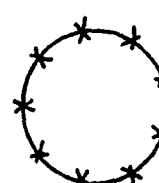
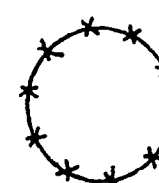
NFS	a	Δ	Pictorial
6	$\frac{\pi}{3}$	$\frac{\pi}{12}$	
7	$\frac{2\pi}{7}$	$\frac{\pi}{14}$	
8	$\frac{\pi}{4}$	$\frac{\pi}{16}$	
9	$\frac{2\pi}{9}$	$\frac{\pi}{18}$	
10	$\frac{\pi}{5}$	$\frac{\pi}{20}$	

Table 1. Matchpoint positions resulting from formulae

$$a = \frac{2\pi}{\text{NFS}} \quad , \quad \Delta = \frac{\pi}{2 \cdot \text{NFS}} \quad .$$

ever, the matchpoint selection technique should provide a rapid convergence of the generalized capacitance matrix to one which is symmetric about the main and cross diagonals.

As indicated above, the number of matchpoints on each conductor surface and each dielectric surface of each wire will be denoted by NFC and NFD, respectively. In addition, the number of matchpoints chosen on each surface will equal the number of Fourier series expansion terms associated with that surface. For the case of two wires, the matchpoint selection technique outlined previously can be shown to yield centro-symmetric generalized capacitance matrices for any value of NFC or NFD. To show this, consider the case of two wires with various matchpoint selections shown in Fig. 3.6. These cases should be sufficiently representative to illustrate the proof.

Note in Fig. 3.6 that when the number of matchpoints on a particular surface is odd, these matchpoints are mirror symmetric about a vertical line X-X' between wire centers. When the number of matchpoints on a particular surface is even, these matchpoints exhibit skew symmetry about the vertical line X-X'. These conclusions can be generalized for two wires and any value of NFC or NFD.

The generalized capacitance matrix for two wires becomes

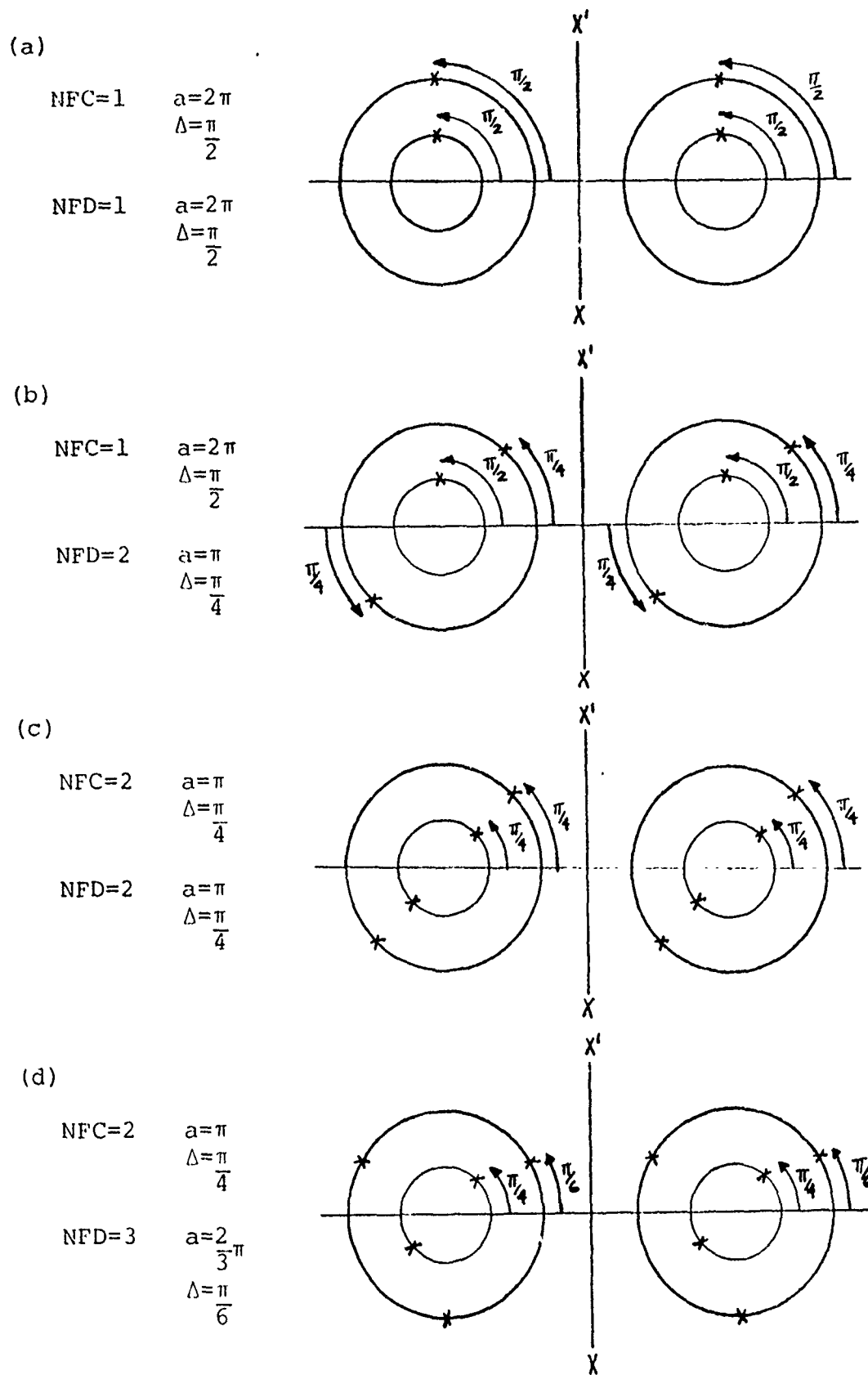


Figure 3.6. Matchpoint selections for two wires.

$$\underline{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} . \quad (3.11)$$

Recall that C_{ij} is the free charge on the i -th conductor with a unit potential applied to the j -th conductor and the remaining conductors at zero potential. Since (1) the matchpoints on corresponding surfaces exhibit either mirror symmetry or skew symmetry with respect to the vertical line $X-X'$, (2) the wires are identical, and (3) the assumed charge distributions are symmetric with respect to a line between centers, then clearly \underline{C} for two wires will exhibit centro-symmetry and

$$\begin{aligned} C_{11} &= C_{22} \\ C_{12} &= C_{21} \end{aligned} . \quad (3.12)$$

This is a direct result of the fact that due to the above observations, the computations performed when the cable is viewed from either end will be identical. Note that for two wires, centro-symmetry automatically guarantees symmetry of about the main diagonal. Computed results illustrating this are given in Table 2 for the cases of $NFC=2$, $NFD=2$ and $NFC=2$, $NFD=3$. Note that the matrices are symmetric with respect to the main and cross diagonals to sixteen digits. All computations were performed on an IBM 370/165 computer in double precision arithmetic.

For cables containing more than two wires, geometrical

Physical conditions:

$r_c = 1.0$
 $r_d = 2.0$
 $sep = 4.0$
 $\epsilon_r = 4.0$

Two wires:
 NFC=2
 NFD=2

$\begin{bmatrix} 23.64630331172406D-12 & -50.61938353186358D-12 \\ -50.61938353186358D-12 & 23.64630331172406D-12 \end{bmatrix}$

Two wires:
 NFC=2
 NFD=3

$\begin{bmatrix} 28.13906061773703D-12 & -55.04463986326089D-12 \\ -55.04463986326089D-12 & 28.13906061773703D-12 \end{bmatrix}$

Five Wires, NFC=3, NFD=7. (elements $\times 10^{-12}$)

$\begin{bmatrix} 48.001873820947 & -43.291394174935 & -4.6562119966386 & -3.3655186155275 & -6.4108937028478 \\ -43.291394174935 & 85.705646411657 & -40.155930238945 & -2.8839728747193 & -3.3654719979811 \\ -4.6562119966386 & -40.155930238945 & 86.008879896960 & -40.156144007001 & -4.6562516882283 \\ -3.3654719979811 & -2.8839728747193 & -40.155930238945 & 85.705646411656 & -43.291394174935 \\ -6.4108937028478 & -3.3655186155275 & -4.6562119966386 & -43.291394174935 & 48.001873820947 \end{bmatrix}$

Table 2. Computed generalized capacitance matrices for two and five wires.

symmetry (i.e., centro-symmetry) of \tilde{C} will again result. This has been verified computationally in numerous cases for cables involving up to twenty wires. However, for more than two wires, this geometrical symmetry does not automatically imply symmetry about the main diagonal. Again, the reason that \tilde{C} will not be symmetric with respect to the main diagonal is that only the approximate charge distributions are obtained. However, this matchpoint selection technique does provide rapid convergence to generalized capacitance matrices which are symmetric about the main and cross-diagonals. A typical result for a five wire cable with $NFC=3$, $NFD=7$ is shown in Table 2.

III.5 Number of Fourier Coefficients and Convergence of the Capacitance Matrix

One final topic to be discussed is the choice of the number of Fourier series coefficients around each boundary which are to yield results to a certain accuracy. This is most important in terms of computation time, since a single additional coefficient increases the size of the \tilde{D} matrix by an order of NW (the number of wires). This is significant since the number of operations (multiplications and divisions) required to invert an $m \times m$ matrix is approximately m^3 . Therefore, the number of operations required to invert \tilde{D} is $(NFC + NFD)^3 (NW)^3$.

A number of sample cases were run to determine worst case needs. The problem considered is illustrated in Fig. 3.7,

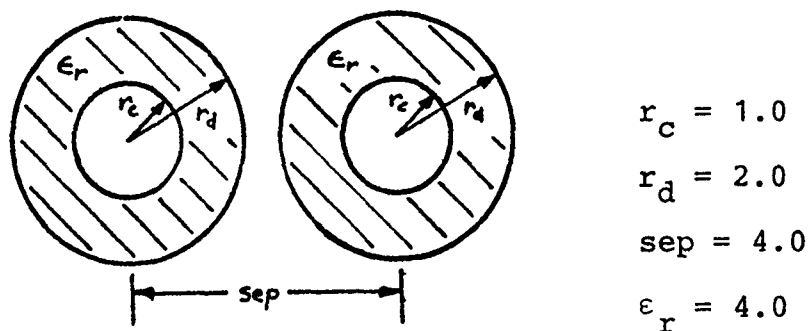


Figure 3.7 Two-wire ribbon cable.

where the dimensions of the cable indicate that the dielectric surfaces are touching. This should correspond to a worst case condition in terms of wire separation. Table 3 shows the calculated capacitance using Eq. (2.5), for various numbers of Fourier coefficients associated with each surface.

The circled items in Table 3 indicate the best result for a particular number of total Fourier coefficients assigned to each wire. The most interesting point is that not as many terms are required around the conductor surface as the dielectric surface. For example, with ten total coefficients, the best organization would use three terms in the representation of the charge distribution around the conductor surface and seven for the dielectric surface.

Table 4(a) indicates similar results for five-wire ribbon cable. Considering the convergence of the $\epsilon_{3,}$ term given in Table 4(a), it is sufficient to assign approximately one-third of the total number of expansion coefficients allotted to the wire to the conductor surface expansion and assign the remaining number to the dielectric surface

# terms around dielectric	# terms around conductor									
	1	2	3	4	5	6	7	8	9	10
1	(27.8)									
2	(33.2)	37.1								
3	(37.3)	(41.6)	42.1							
4	39.7	(44.4)	45.0	45.0						
5	41.0	(46.1)	46.7	46.8	46.8					
6	41.7	(47.1)	(47.7)	47.8	47.8	47.8				
7	42.1	(47.7)	(48.4)	48.4	48.5		48.5			
8	42.4	48.1	(48.7)	48.8	48.8			48.8		
9	42.6	48.3	(49.0)	(49.1)	49.1				49.1	
10	42.7	48.4	(49.1)	(49.2)	49.2					49.2
11			(49.2)	(49.3)						
12			(49.3)							

Physical conditions: $r_c = 1.0$, $r_d = 2.0$, $\text{sep} = 4.0$, $\epsilon_r = 4.0$.

Table 3. Capacitance between two dielectric-coated conductors for various numbers of Fourier terms per boundary.
(All entries $\times 10^{-12}$.)

# terms around dielectric	# terms around conductor							10
	1	2	3	4	5	6	SS	
1	(46.2)							
2	(56.3)	63.5						
3	(65.1)	(72.4)	73.4					
4	70.5	(78.1)	79.4	79.5				
5	73.0	(81.3)	82.7	82.8	82.8			
6	74.7	(83.5)	(84.8)	85.0	85.0			
7	75.6	84.6	(86.0)	86.2	86.2			
8	76.2	85.3	(86.8)	87.0	87.0			
9	76.6	85.7	(87.3)	87.4	87.4			
10	76.8	86.0	(87.6)	87.7	87.8			87.8
11			(87.8)	(87.9)	87.9			
12			(87.9)	(88.1)	(88.1)			
13			88.0	(88.1)	(88.2)			
14			88.0	(88.2)	88.2			

Physical conditions:

$$r_c=1.0, r_d=2.0, \text{sep}=4.0, \epsilon_r=4.0$$

Table 4(a). Center element, $C(3,3)$, in the generalized capacitance matrix of a five-wire ribbon cable for various numbers of Fourier terms per boundary. (All entries $\times 10^{-12}$.)

expansion. In applying these results to transmission line analysis, the transmission line capacitance matrix, \underline{C} , is needed. The derivation of \underline{C} from the generalized capacitance matrix is shown in Chapter V, and the C_{11} term for the five wire ribbon cable is shown in Table 4(b). Again, it is sufficient to proportion the total number of expansion coefficients allotted to a wire by assigning approximately one-third to the conductor surface. This allotment criterion has yielded the same convergence results for all computed cases which were examined and should serve as a general rule for users of this program.

A general rule for the total number of coefficients to be allotted to each wire cannot be as easily obtained since the convergence depends upon the relative separation of the wires; for larger wire separations, a smaller number of total expansion terms can be used. However, a worst case rule can be cited. Numerous examples have been run involving up to twenty wires in which the dielectric surfaces are touching (a worst case and generally typical situation for ribbon cables). The dielectric constant is assumed to be 4 (a representative upper bound on typically used insulation materials). For all of these cases, accuracy to within approximately five digits can be obtained by using a total of ten coefficients per each wire. With the above allotment rule, the recommendation is to use a total of ten terms for each wire and assign three to the conductor surface and seven to the dielectric surface.

# terms around dielectric	# terms around conductor									
	1	2	3	4	5	6	SS		10	
1	(46.6)									
2	(56.6)	63.7								
3	(65.4)	(72.6)	73.6							
4	70.8	(78.3)	79.6	79.7						
5	73.4	(81.6)	82.9	83.0	83.0					
6	75.1	(83.7)	(85.1)	85.2	85.2					
7	75.9	84.8	(86.2)	86.4	86.4					
8	76.6	85.5	(87.0)	87.2	87.2					
9	76.9	85.9	(87.5)	87.6	87.7					
10	77.1	86.2	(87.8)	(88.0)	88.0				88.0	
11			(88.0)	(88.1)	88.2					
12			(88.1)	(88.3)	(88.3)					
13			88.2	(88.3)	(88.4)					
14			88.2	(88.4)	88.4					

Physical conditions:

$$r_c=1.0, r_d=2.0, \text{sep}=4.0, \epsilon_r=4.0$$

Table 4(b). C_{11} element in the transmission line capacitance matrix (to be described in Chapter V) of a five-wire ribbon cable for various numbers of Fourier terms per boundary.
(All entries $\times 10^{-12}$.)

CHAPTER IV

ALTERNATE METHODS OF SOLUTION

IV.1 Introduction

A numerical method for determining the generalized capacitance matrix for ribbon cables was developed in Chapter II. Criteria for the selection of the matchpoints were given in Chapter III.

One difficulty with this technique is that for large numbers of wires, a reasonably accurate \tilde{C} matrix requires that the \tilde{D} matrix of Eq. (2.23) be of a very large order. As indicated previously, the number of operations required to obtain \tilde{D}^{-1} is on the order of $(NFC+NFD)^3(NW)^3$ where NW is the number of wires in the system and NFC and NFD are the number of matchpoints (expansion terms) associated with the conductor and dielectric boundaries, respectively.

The solution of the matrix equation (2.28) to yield the generalized capacitance matrix could be obtained in much less time if there were a large number of zero elements in the \tilde{D} matrix. There exist many routines which take advantage of this "sparseness" of the coefficient matrix to optimize solution of the equations.

In this chapter, two methods which result in sparse \tilde{D} matrices will be discussed. One is a "near-neighbor"

technique in which only close wires are assumed to affect one another. The other is a "term-dropping" technique based on the relative separation of the wires. Neither of these techniques proved to yield valid results.

IV.2 The Near-Neighbor Technique

In this technique, it is assumed that the potential and displacement vector at the matchpoints on a particular wire have contributions from the charge distributions on only the wires closest to it. A term will be defined to describe this closeness. Consider the multiwire ribbon cable shown in Fig. 4.1. The term "first near-neighbors" will refer to the wires directly adjacent to the match wire, "second near-neighbors" will refer to the wires separated by one wire from the match wire, etc., as noted in Fig. 4.1.

When calculating a particular row of the \tilde{D} matrix, the terms corresponding to a particular number of near-neighbors will be included; the rest will be assumed to be small and

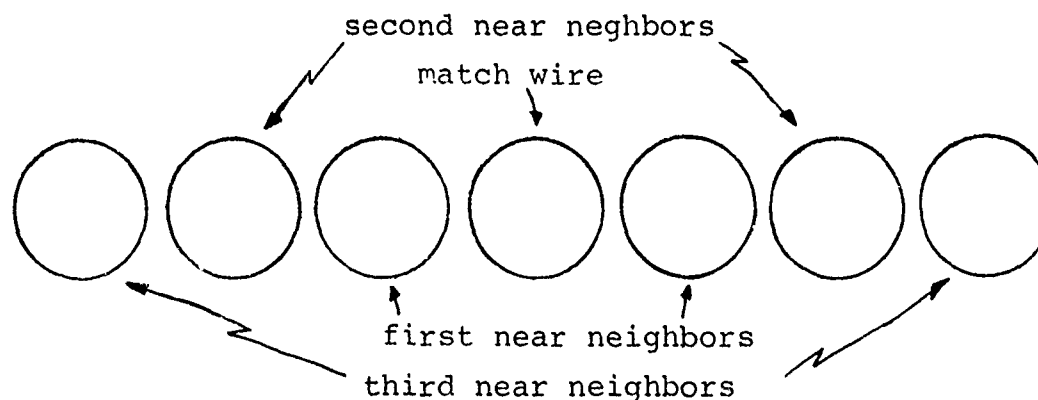


Figure 4.1. Near neighbors of a multiwire ribbon cable.

will be set equal to zero. The more terms that are neglected, the more sparse the D matrix becomes.

The goal was to find an optimum number of near-neighbors to include such that the results are valid, and yet the D matrix is sparse enough to make solution worthwhile through the sparse matrix techniques.

Results indicate, however, that a near-neighbor technique is not valid. Shown in Table 5 is a comparison of several terms of the generalized capacitance matrix for the exact (no assumptions) and the near-neighbor methods. Not only are the magnitudes of the particular elements quite different, but the required sign pattern of the generalized capacitance matrix (positive diagonal elements, negative off-diagonal elements) has been altered. Similar results occurred for the case of ten wires with eight near-neighbors included (only the effects of the end wires on one another were neglected). It would seem from these results that this method is unusable.

Careful investigation of the physical conditions of the previous method shows that an assumption which was made is invalid. In electromagnetics, it is valid to assume that when two surface charge distributions are widely separated, the distributions themselves are approximately constant around the periphery of the charged surfaces.

Thus, assuming no effect on the potential or displacement vector at a matchpoint on one wire due to the charge distribution on another wire when the wires are widely

Elements of the Generalized Capacitance
Matrix for a Three-wire Ribbon Cable.

(elements $\times 10^{-12}$, pF/M)

Element	Control	First Near-neighbor
$C(1,1)$	36.44	-25.80
$C(1,2)$	-39.43	-22.96
$C(1,3)$	-13.73	40.20
$C(2,2)$	70.62	84.04
$C(2,3)$	-39.43	-22.96
$C(3,3)$	36.44	-25.80

Physical conditions: $r_c = 1.0$

$r_d = 2.0$

sep=4.0

$\epsilon_r = 3.0$

Ten Fourier coefficients chosen
around each boundary.

Table 5. Comparison of results using the
near-neighbor technique to
results from the control case
(no dropping of terms).

separated, as in the method above, is clearly not valid. However, assuming that this effect would be due to a constant distribution would seem more logical. This is the idea behind the second method which will be described in the next section.

IV.3 The Term-Dropping Technique

The results of the previous section indicate that the "near-neighbor" technique does not adequately describe the physical conditions of the ribbon cable. This method, then, will not be useful in reducing the computation time required to find the generalized capacitance matrix. The results did, however, point to another possible technique which will be described in this section.

The "term-dropping" technique is based on the assumption that the effects between two widely separated surface charge distributions are produced mainly by the average value of charge around the periphery of the charged surfaces. In this report, the surface charge distributions are represented by a Fourier series. Thus, the total contribution to the potential or displacement vector at a match point on the match wire should be adequately represented by a charge distribution on the source wire which has fewer number of Fourier series coefficients as the separation between the wires increases. An example of the "term-dropping" technique is illustrated in Fig. 4.2.

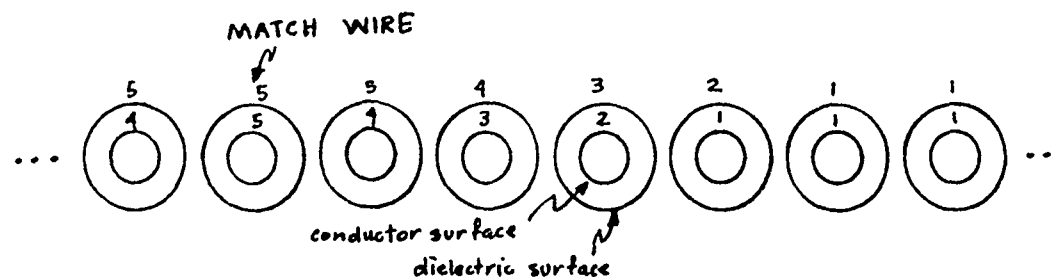
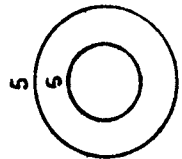
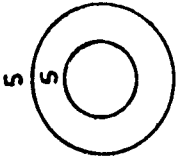
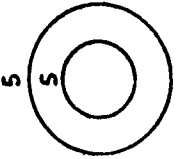
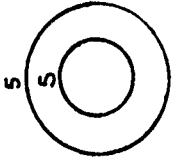
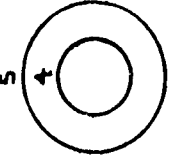
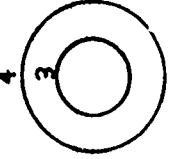
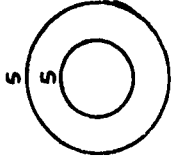
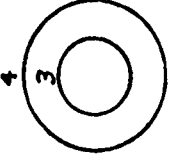
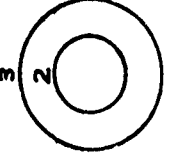
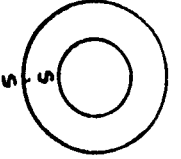
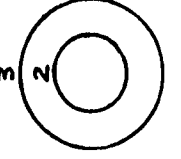
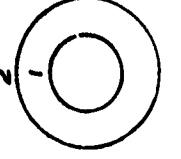


Figure 4.2. Multiwire flat-pack cable illustrating the "term-dropping" technique. (Five Fourier coefficients per boundary.)

The second wire from the left in Fig. 4.2 is considered the match wire; that is, the contributions to the potential and displacement vector at matchpoints on the surfaces of the match wire due to all the wires are to be determined. The numbers on the surfaces indicate the number of components of the Fourier series representing the charge on that surface which will be considered to effect the potential and electric field at the match wire. For example, the number 3 will indicate that only the expansion terms 1, $\cos\theta$, and $\cos2\theta$ are used to represent the distribution on the particular surface; the number 6 indicates that the terms 1, $\cos\theta$, $\cos2\theta$, $\cos3\theta$, $\cos4\theta$, and $\cos5\theta$ are retained.

A three wire flat-pack cable with conductor radius of 1.0, dielectric radius of 2.0, separation of 4.0, and relative dielectric constant of 3.0, was analyzed using this technique. Results from the term-dropping method are compared to the control case (no dropping of terms) in Table 6.

The data in Table 6 indicate that accuracy is fairly well maintained for all cases. In the case yielding 39.44% sparsity (the percentage of the total elements in D which

Type	Distribution	Typical terms $\times 10^{-12}$	Sparsity & Accuracy
Control	  	$e(1,1) = 35.2894400$ $e(1,2) = -38.2562409$ $e(1,3) = -13.7449299$ $e(2,2) = 68.2786647$	900 elements 15 zeros <u>1.67% sparse</u>
Terms less than 10^{-4} dropped	  	$e(1,1) = 35.2894800$ $e(1,2) = -38.2588920$ $e(1,3) = -13.7422123$ $e(2,2) = 68.2849897$	900 elements 115 zeros <u>12.8% sparse</u> 4 digit accuracy
Terms less than 10^{-3} dropped	  	$e(1,1) = 35.3077153$ $e(1,2) = -38.3264668$ $e(1,3) = -13.6914884$ $e(2,2) = 68.4380311$	900 elements 235 zeros <u>26.1% sparse</u> 2.5 digit accuracy
Terms less than 10^{-2} dropped	  	$e(1,1) = 34.7760576$ $e(1,2) = -38.4286168$ $e(1,3) = -12.9417964$ $e(2,2) = 69.1768575$	900 elements 355 zeros <u>39.4% sparse</u> 1.5 digit accuracy

Three-wire ribbon cable.

Physical conditions:

$$r_c = 1.0, r_d = 2.0, \text{ sep} = 4.0, \epsilon_r = 3.0.$$

Table 6. Comparison of results from the control case (no dropping) to results from the term-dropping technique.

are zero), the error in the elements of the generalized capacitance matrix is at most 5.84%. Since this term-dropping technique appears to work so well for three wires, extension to more than three wires is called for. The term-dropping technique was applied to the cases of 4, 5, and 6 wire ribbon cables, with the same wire characteristics, dimensions and separations, as the three wire cable. The data in Table 7A is a comparison of representative elements of the generalized capacitance matrix which were computed using this technique to the same elements from the control case (no terms dropped). These elements are chosen to indicate the relative behavior of all the elements of the matrix.

For the case of 4 wires, the elements of \tilde{C} agree to the control case to within approximately 5% when elements of \tilde{D} less than 10^{-3} are set to zero. However, for the 5 and 6 wire cables, with elements in \tilde{D} less than 10^{-4} set to zero, the error has increased to approximately 10% and 33% respectively.

The error can be corrected by adding more Fourier coefficients to the surface charge representations, while simultaneously changing the threshold for setting elements to zero to those less than 10^{-5} . This new threshold can be illustrated by showing the number of Fourier series coefficients representing each surface charge distribution which will contribute to the potential and field equations on the match wire, as in Fig. 4.3.

Representative Elements of the Generalized Capacitance
Matrix with 5 Fourier Coefficients per Boundary

(Elements $\times 10^{-12}$.)

Number of Wires	Element	No terms dropped	Terms dropped:		
			10^{-4}	10^{-3}	10^{-2}
4	$C(1,4)$	-8.63927	-8.62931	-8.17036	-6.99345
5	$C(1,4)$	-3.37309	-3.71094	-4.26695	-2.77220
	$C(1,5)$	-6.30418	-5.97030	-4.92757	-4.93595
6	$C(1,5)$	-2.41456	-3.20696	-2.98949	-.454844
	$C(1,6)$	-4.94645	-3.82304	-3.70573	-5.32934

Table 7A. Comparison of results from term-dropping technique for 4, 5, and 6-wire ribbon cables with 5 Fourier coefficients per boundary.

Representative Elements of the Generalized Capacitance
Matrix with 6 Fourier Coefficients per Boundary

(Elements $\times 10^{-12}$.)

Element	No terms dropped	Terms dropped:			
		10^{-5}	10^{-4}	10^{-3}	10^{-2}
$C(1,5)$	-2.41453	-2.41808	-2.68969	-3.25143	-2.09194
$C(1,6)$	-4.94634	-4.94287	-4.68013	-3.76534	-3.64053

Table 7B. Comparison of results from term-dropping technique for a 6-wire ribbon cable with 6 Fourier coefficients per boundary.

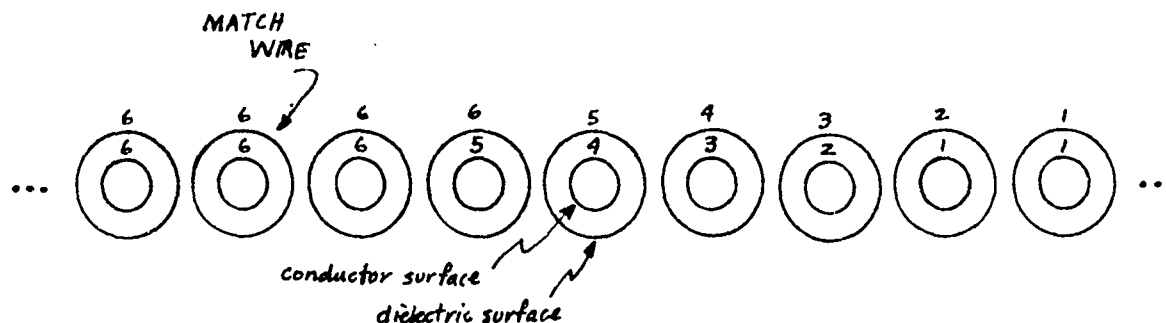


Fig. 4.3. Modified term-dropping technique with 6 Fourier coefficients representing each surface charge.

These two changes were incorporated into the analysis of the 6 wire ribbon cable. The results are shown in Table 7B. It is evident from these data that the combination of increasing the number of Fourier coefficients used to represent the surface charges and changing the dropping threshold produces accurate results. But from a computational standpoint, this also increases the size of the \underline{D} matrix from a 60x60 matrix to a 72x72 matrix, while simultaneously reducing the sparsity of the \underline{D} matrix. Thus, sparse matrix techniques would not be useful for these problems.

The term-dropping technique, then, does not appear to be particularly useful for two reasons:

- (1) As the number of wires in the cable increases, the size of the \underline{D} matrix must be increased to allow the use of more Fourier coefficients in the representations for the surface charge distributions. This increase in size will increase

computation time significantly.

- (2) As the number of wires in the cable increases, the \tilde{D} matrix must be made less sparse to make the results accurate.

IV.4 Conclusions

The results of the previous two sections indicate that neither the near-neighbor technique nor the term-dropping technique yielded sufficiently valid results to be useful in determining the generalized capacitance matrix of ribbon cables.

An efficient method for finding the inverse of a block Toeplitz matrix would be useful, however, since the \tilde{D} matrix is in this form, as shown in Eq. (2.28) [14,17]. Methods for inversion of block Toeplitz matrices have been developed, but all require further constraints on the blocks. Sinnott's inversion algorithm [17] requires that the entire block Toeplitz matrix, as well as the block submatrices, be symmetric. Cramer's algorithm [14] requires that the block submatrices be scalar Toeplitz matrices. It does not appear that the requirements of either of these methods can be met with the structure of the \tilde{D} matrix for ribbon cables.

A method applicable to this problem yielding one-half of \tilde{D}^{-1} will be shown later in Chapter VI. This method will result in a time savings of approximately five-eighths over full inversion.

CHAPTER V

THE TRANSMISSION LINE CAPACITANCE AND INDUCTANCE MATRICES

V.1 Introduction

In Chapter II, a method for determining the generalized capacitance matrix of a ribbon cable with n wires was shown. Computation of crosstalk in this type of cable can be accomplished through a solution of the multiconductor transmission line equations [2]. This method utilizes the per-unit-length transmission line capacitance and inductance matrices. The transmission line capacitance matrix differs from the generalized capacitance matrix in that a particular conductor in the cable is chosen as a "reference conductor", and all cable voltages are referenced to this conductor.

In Section V.2, a technique for computing the transmission line capacitance matrix from the generalized capacitance matrix is given. In Section V.3, a formula yielding an approximate transmission line inductance matrix for ribbon cables is developed. Section V.4 deals with setting up the transmission line equations for cases involving multiple reference conductors in ribbon cables.

V.2 Determining the Transmission Line Capacitance Matrix From the Generalized Capacitance Matrix

The "generalized" capacitance matrix $\underline{\tilde{C}}$ is defined by the matrix equation

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{nf} \end{bmatrix} = \underline{\tilde{C}} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \quad (5.1)$$

for an n-wire cable, with q_{if} indicating free charge on conductor i, and ϕ_i indicating the potential on conductor i with respect to some arbitrary reference point.

For the purpose of describing this method, the n-th conductor in the cable will be chosen as the reference for the transmission line voltages, defined by

$$V_i = (\phi_i - \phi_n) , \quad i = 1, 2, \dots, n-1 \quad . \quad (5.2)$$

The transmission line capacitance matrix, \underline{C} , is of order (n-1) and is defined by

$$\begin{bmatrix} q_{1f} \\ \vdots \\ q_{(n-1)f} \end{bmatrix} = \begin{bmatrix} C_{11} & \cdots & C_{1(n-1)} \\ \vdots & & \vdots \\ C_{(n-1)1} & & C_{(n-1)(n-1)} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_{(n-1)} \end{bmatrix} \quad (5.3)$$

To determine the transmission line capacitance matrix, \underline{C} ,

from the generalized capacitance matrix, ϵ , we require the per-unit-length free charge on the reference conductor to satisfy the constraint

$$q_{nf} = - \sum_{i=1}^{n-1} q_{if} \quad (5.4)$$

Thus Eq. (5.1) can be written as

$$\begin{aligned} q_{1f} &= \epsilon_{11}V_1 + \epsilon_{12}V_2 + \dots + \epsilon_{1n}V_n + \left(\sum_{m=1}^n \epsilon_{1m}\right)\phi_n \\ q_{2f} &= \epsilon_{21}V_1 + \epsilon_{22}V_2 + \dots + \epsilon_{2n}V_n + \left(\sum_{m=1}^n \epsilon_{2m}\right)\phi_n \\ &\vdots \\ - \sum_{i=1}^{n-1} q_{if} &= \epsilon_{n1}V_1 + \epsilon_{n2}V_2 + \dots + \epsilon_{nn}V_n + \left(\sum_{m=1}^n \epsilon_{nm}\right)\phi_n \end{aligned} \quad (5.5)$$

Adding all equations in Eq. (5.5) together results in

$$\begin{aligned} 0 &= \left(\sum_{m=1}^n \epsilon_{m1}\right)V_1 + \left(\sum_{m=1}^n \epsilon_{m2}\right)V_2 + \dots + \left(\sum_{m=1}^n \epsilon_{mn}\right)V_n \\ &\quad + \left(\sum_{m=1}^n \epsilon_{1m}\right)\phi_n + \sum_{m=1}^n \epsilon_{2m}\phi_n + \dots + \sum_{m=1}^n \epsilon_{nm}\phi_n \end{aligned} \quad (5.6)$$

Solving Eq. (5.6) for ϕ_n yields

$$\phi_n = - \frac{\sum_{k=1}^n \left[\left(\sum_{m=1}^n \epsilon_{mk}\right)V_k \right]}{\sum_{l=1}^n \left[\sum_{m=1}^n \epsilon_{lm} \right]} \quad (5.7)$$

Substituting Eq. (5.7) into the first (n-1) equations in Eq. (5.5) yields the entries in the transmission line capacitance matrix as

$$[C]_{ij} = C_{ij} = c_{ij} - \frac{\left[\sum_{m=1}^n c_{im} \right] \left[\sum_{m=1}^n c_{mj} \right]}{\sum_{l=1}^n \left[\sum_{m=1}^n c_{lm} \right]} \quad (5.8)$$

for $i, j = 1, \dots, (n-1)$. Eq. (5.8) can be rewritten as

$$C_{ij} = \frac{c_{ij} \left\{ \sum_{l=1}^n \left[\sum_{m=1}^n c_{lm} \right] \right\} - \left\{ \sum_{m=1}^n c_{im} \right\} \left[\sum_{m=1}^n c_{mj} \right]}{\sum_{l=1}^n \left[\sum_{m=1}^n c_{lm} \right]} \quad (5.9)$$

Note that the denominator of Eq. (5.9) is simply the sum of all elements of the generalized capacitance matrix, \tilde{C} . The second term of the numerator can be expanded by deleting the products with c_{ij} yielding for the numerator of Eq. (5.9)

$$\begin{aligned} c_{ij} \left\{ \sum_{l=1}^n \left[\sum_{m=1}^n c_{lm} \right] \right\} - c_{ij} \left(\sum_{\substack{m=1 \\ m \neq i}}^n c_{mj} \right) - c_{ij} \left(\sum_{\substack{m=1 \\ m \neq j}}^n c_{im} \right) - c_{ij}^2 \\ - \left(\sum_{\substack{m=1 \\ m \neq j}}^n c_{im} \right) \left(\sum_{\substack{m=1 \\ m \neq i}}^n c_{mj} \right) \end{aligned} \quad (5.10)$$

Combining the first four terms of Eq. (5.10), the numerator of Eq. (5.9) becomes

$$C_{ij} \left\{ \sum_{l=1}^n \left[\sum_{\substack{m=1 \\ m \neq i}}^n C_{lm} \right] - \sum_{\substack{m=1 \\ m \neq i}}^n C_{mj} - \sum_{\substack{m=1 \\ m \neq j}}^n C_{im} - C_{ij} \right\} - \left(\sum_{\substack{m=1 \\ m \neq j}}^n C_{im} \right) \left(\sum_{\substack{m=1 \\ m \neq i}}^n C_{mj} \right) \quad (5.11)$$

or

$$C_{ij} \left\{ \sum_{\substack{l=1 \\ l \neq i}}^n \left[\sum_{\substack{m=1 \\ m \neq j}}^n C_{lm} \right] \right\} - \left(\sum_{\substack{m=1 \\ m \neq j}}^n C_{im} \right) \left(\sum_{\substack{m=1 \\ m \neq i}}^n C_{mj} \right) \quad (5.12)$$

The first term in Eq. (5.12) is the term C_{ij} multiplied by the sum of all the terms in \tilde{C} except for those in the i -th row and j -th column. The second term in Eq. (5.12) is the product of the sum of the elements in the i -th row of \tilde{C} (except C_{ij}) with the sum of the elements in the j -th column of \tilde{C} (except C_{ij}).

The above results yield a simple procedure for determining the transmission line capacitance matrix from the generalized capacitance matrix with the n -th conductor chosen as the reference conductor for the transmission line voltages.

Rewriting Eq. (5.9) by inserting the numerator of Eq. (5.12) yields

$$C_{ij} = \frac{C_{ij} \left\{ \sum_{l=1, l \neq i}^n \left[\sum_{m=1, m \neq j}^n C_{lm} \right] \right\} - \left(\sum_{m=1, m \neq j}^n C_{im} \right) \left(\sum_{m=1, m \neq i}^n C_{mj} \right)}{\sum_{l=1}^n \left[\sum_{m=1}^n C_{lm} \right]} \quad (5.13)$$

for $i, j = 1, \dots, (n-1)$. Thus, given the generalized capacitance matrix \tilde{C} , as shown in Eq. (5.1), to find the term C_{ij} of the transmission line capacitance matrix C , in which the n -th conductor is chosen as reference for the transmission line voltages,

- 1) multiply \tilde{C}_{ij} by the sum of the terms of the matrix \tilde{C} with the i -th row and j -th column deleted,
- 2) subtract the product of the sum of all terms of the i -th row (except \tilde{C}_{ij}) with the sum of the terms of the j -th column (except \tilde{C}_{ij}), and
- 3) divide by the sum of all terms in the generalized capacitance matrix.

$$\begin{array}{c}
 \text{j-th col.} \nearrow \\
 \begin{bmatrix}
 \tilde{C}_{11} & \tilde{C}_{12} & \dots & \tilde{C}_{1j} & \dots & \tilde{C}_{1n} \\
 \tilde{C}_{21} & \tilde{C}_{22} & \dots & \tilde{C}_{2j} & \dots & \tilde{C}_{2n} \\
 \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
 \tilde{C}_{i1} & \tilde{C}_{i2} & & \tilde{C}_{ij} & & \tilde{C}_{in} \\
 \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
 \tilde{C}_{n1} & \tilde{C}_{n2} & \dots & \tilde{C}_{nj} & \dots & \tilde{C}_{nn}
 \end{bmatrix} \\
 \text{i-th row} \rightarrow
 \end{array} \quad (5.14)$$

The above method should be easy to do by hand especially for a small number of wires. For computer implementation, however, the form of Eq. (5.8) is used as shown below:

$$C_{ij} = C_{ij} - \frac{\left(\sum_{m=1}^n C_{im} \right) \left(\sum_{m=1}^n C_{mj} \right)}{\sum_{l=1}^n \left[\sum_{m=1}^n C_{lm} \right]} \quad (5.15)$$

for $i, j = 1, \dots, (n-1)$. The procedure is as follows:

- 1) Sum the elements in the n -th row of \tilde{C} . This sum will be used to find the sum of the elements in \tilde{C} .
- 2) For $i=1, \dots, (n-1)$
 - a) Sum the elements in the i -th row of \tilde{C} . Add to the matrix sum. Insert this sum into the elements of the i -th row of the \tilde{C} matrix.
 - b) Sum the elements in the i -th column of \tilde{C} . Store these sums.
- 3) For $i=1, \dots, (n-1)$
 - a) Multiply C_{ij} by the sum of column j , divide by the matrix sum, then subtract from \tilde{C}_{ij} and insert in C_{ij} .

This procedure was developed for obtaining the per-unit-length transmission line capacitance matrix for an n -wire cable with the n -th wire chosen as reference for the transmission line voltages. Clearly, this is readily adaptable to the case where any other wire in the cable is chosen as a reference for the transmission line voltages. To show this, it is sufficient to note that the rows and columns of the generalized capacitance matrix may be exchanged to obtain a representation with the per-unit-length free charge on the reference conductor, q_{rf} , as the last entry in the vector \underline{q}_f and the potential of the reference conductor, ϕ_r ,

as the last entry in the vector $\underline{\phi}$. The particular ordering and structure of \underline{C} with respect to the conductor numbering system used in the computer program will be discussed in Section VII.4.

V.3 The Per-Unit-Length Transmission Line Inductance Matrix for Ribbon Cables

In order to solve the transmission line equations for a cable system, the per-unit-length transmission line capacitance and inductance matrices are needed. In the previous section a method was developed yielding the transmission line capacitance matrix from the generalized capacitance matrix computed by the method described in Chapter II. The per-unit-length transmission line inductance matrix is a function of the wire orientation and is independent of the characteristics of the dielectric insulation materials. This is true since the permeability of dielectrics is typically that of free space. Thus, the inductance matrix for a system of insulated wires is the same as that for a system of uninsulated wires with an identical wire orientation.

The per-unit-length transmission line inductance matrix for a system of uninsulated conductors must obey the equation [15]

$$\underline{L} = \mu_v \epsilon_v \underline{C}^{-1} \quad , \quad (5.16)$$

where μ_v is the permeability of free space, ϵ_v is the

permittivity of free space, and \tilde{C}_0^{-1} is the inverse of the per-unit-length transmission line capacitance matrix with the dielectrics removed.

Therefore, given the transmission line capacitance matrix for the ribbon cable with the dielectric insulation ignored, the transmission line inductance matrix can be easily found.

For most ribbon cables, the ratio of conductor radius to conductor separation usually is greater than 4. (If the insulation thickness is equal to the wire radius and the insulations were touching, the ratio would equal 4.) In many instances it may be sufficient to find an approximate form of the transmission line inductance matrix instead of recomputing the transmission line capacitance matrix with the dielectrics removed.

In developing this approximation, consider the case of n wires, in which the n -th wire is chosen as reference.

Fig. 5.1 illustrates conditions on two typical conductors, i

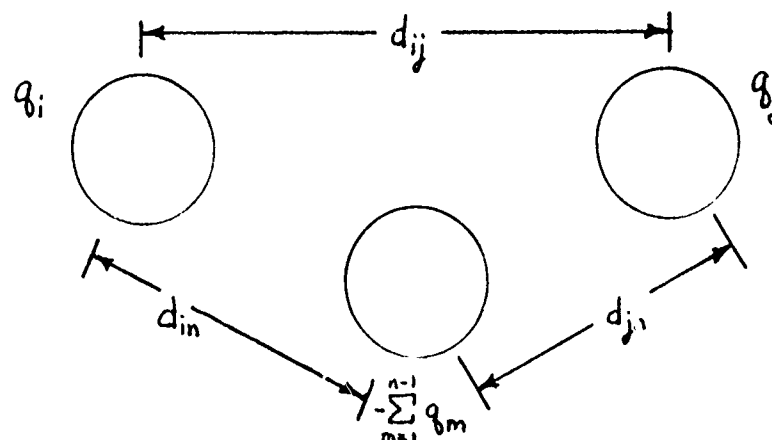


Figure 5.1. Two typical conductors and the reference conductor.

and j , as well as on the reference conductor, labeled n . The per-unit-length charge on conductor i (which for this uninsulated case will be entirely free charge) is q_i , d_{ij} is the distance between the centers of wires i and j , and V_i is the potential of conductor i with respect to the reference. The relationship between the conductor potentials, V , and the charge on the conductors, q , is represented by

$$\begin{bmatrix} V_1 \\ \vdots \\ V_{(n-1)} \end{bmatrix} = \underbrace{\begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{(n-1)1} & \cdots & C_{(n-1)(n-1)} \end{bmatrix}}_{\underline{C}_0}^{-1} \begin{bmatrix} q_1 \\ \vdots \\ q_{(n-1)} \end{bmatrix} \quad (5.17)$$

Individual elements of \underline{C}_0^{-1} can be found from the conductor potentials and per-unit-length charges by the equation

$$[\underline{C}_0^{-1}]_{ij} = \left. \frac{V_i}{q_j} \right|_{\substack{q_m=0 \\ m \neq j}} \quad m = 1, \dots, (n-1) \quad (5.18)$$

The generalized capacitance matrix for this case can be found by using the results of Chapter II. Since the conductors are widely spaced, the match points can be chosen at the center of the conductors with constant charge distributions assumed on the conductors. However, when deter-

mining the potential of a wire due to its own charge distribution, the matchpoint will be chosen on the surface of the conductor. Then, from equations (2.12) and (2.14), there result n equations relating the per-unit-length charge distributions, σ_{i0} , and potentials, ϕ_i , in the form

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} = \frac{-1}{\epsilon_v} \begin{bmatrix} r_{c1} \ln(r_{c1}) & r_{c2} \ln(d_{21}) & r_{c3} \ln(d_{31}) & \cdots \\ r_{c1} \ln(d_{21}) & r_{c2} \ln(r_{c2}) & r_{c3} \ln(d_{32}) & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ r_{cn} \ln(d_{n1}) & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} \sigma_{10} \\ \sigma_{20} \\ \vdots \\ \sigma_{n0} \end{bmatrix} \quad (5.19)$$

The potential of any conductor can then be written as a function of the per-unit-length charges q_i . For example, the first equation in (5.19) can be written as

$$\phi_1 = \frac{-1}{2\pi\epsilon_v} \left\{ 2\pi r_{c1} \sigma_{10} \ln(r_{c1}) + 2\pi r_{c2} \sigma_{20} \ln(d_{21}) + \dots \right\} \quad (5.20)$$

where $q_i = 2\pi r_{ci} \sigma_{i0}$ for $i = 1, 2, \dots, n$. Then, equation (5.19) becomes:

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} = \frac{-1}{2\pi\epsilon_v} \begin{bmatrix} \ln(r_{c1}) & \ln(d_{21}) & \cdots \\ \ln(d_{21}) & \ln(r_{c2}) & \cdots \\ \vdots & \vdots & \vdots \\ \ln(d_{n1}) & \ln(d_{n2}) & \cdots \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} \quad (5.21)$$

Now, conductor n can be chosen as reference and the potential, V_i , between conductor i and the reference conductor defined as

$$V_i = \phi_i - \phi_n \quad . \quad (5.22)$$

According to equation (5.21),

$$\phi_n = \frac{-1}{2\pi\epsilon_v} \left\{ \ln(d_{n1})q_1 + \dots + \ln(d_{ni})q_i + \dots + \ln(d_{nj})q_j + \dots + \ln(r_{cn})q_n \right\} \quad (5.23a)$$

and

$$\phi_i = \frac{-1}{2\pi\epsilon_v} \left\{ \ln(d_{i1})q_1 + \dots + \ln(r_{ci})q_i + \dots + \ln(d_{ij})q_j + \dots + \ln(d_{ni})q_n \right\} \quad . \quad (5.23b)$$

Then,

$$\begin{aligned} V_i &= \phi_i - \phi_n \\ &= \frac{-1}{2\pi\epsilon_v} \left\{ \ln\left(\frac{d_{i1}}{d_{n1}}\right)q_1 + \dots + \ln\left(\frac{r_{ci}}{d_{ni}}\right)q_i + \dots + \ln\left(\frac{d_{ij}}{d_{nj}}\right)q_j + \dots + \ln\left(\frac{d_{ni}}{r_{cn}}\right)q_n \right\} \quad . \end{aligned} \quad (5.24)$$

The requirement that the total charge on the reference conductor (conductor n) is minus the sum of the total charges on all the other conductors can again be invoked; that is,

$$q_n = - \sum_{i=1}^{n-1} q_i \quad . \quad (5.25)$$

Rewriting (5.24) by including (5.25), yields

$$V_i = \frac{-1}{2\pi\epsilon_v} \left\{ \dots + \ln\left(\frac{r_{cn}r_{ci}}{(d_{ni})^2}\right)q_i + \dots + \ln\left(\frac{r_{cn}d_{ij}}{d_{ni}d_{nj}}\right)q_j + \dots \right\} \quad (5.26)$$

Then, from equations (5.18) and (5.26), individual elements of \tilde{C}_0^{-1} are

$$[\tilde{C}_0^{-1}]_{ij} = \begin{cases} \frac{1}{2\pi\epsilon_v} \ln \left[\frac{(d_{ni})^2}{r_{cn} r_{ci}} \right] & \text{for } i=j \\ \frac{1}{2\pi\epsilon_v} \ln \left[\frac{d_{ni} d_{nj}}{r_{cn} d_{ij}} \right] & \text{for } i \neq j \end{cases} \quad (5.27)$$

and $i, j = 1, 2, \dots, (n-1)$.

Finally, combining equations (5.16) and (5.27), the elements of an approximation to the per-unit-length transmission-line inductance matrix with conductor n chosen as reference are

$$[\tilde{L}]_{ij} = \begin{cases} \frac{\mu_v}{2\pi} \ln \left[\frac{(d_{ni})^2}{r_{cn} r_{ci}} \right] & \text{for } i=j \\ \frac{\mu_v}{2\pi} \ln \left[\frac{d_{ni} d_{nj}}{r_{cn} d_{ij}} \right] & \text{for } i \neq j \end{cases} \quad (5.28)$$

V.4 Applications to the Analysis of Multiple Reference Conductor Systems

Computer and other digital system interconnections utilize a wire assignment in ribbon cables where many wires are reference conductors. The most common method is the ground-signal-ground technique, where the end wires and every other wire is a reference conductor, so that the signal wires are separated from each other by a reference conductor.

The transmission line equations can be used to solve

the multiple reference conductor problem by obtaining \underline{C} and \underline{L} for the cable with one of the grounded conductors chosen as reference. Then, the terminal impedance matrices, \underline{Z} , can be written by considering grounded wires as zero terminal impedances [2, 13]. A typical example is illustrated in Fig. 5.2.

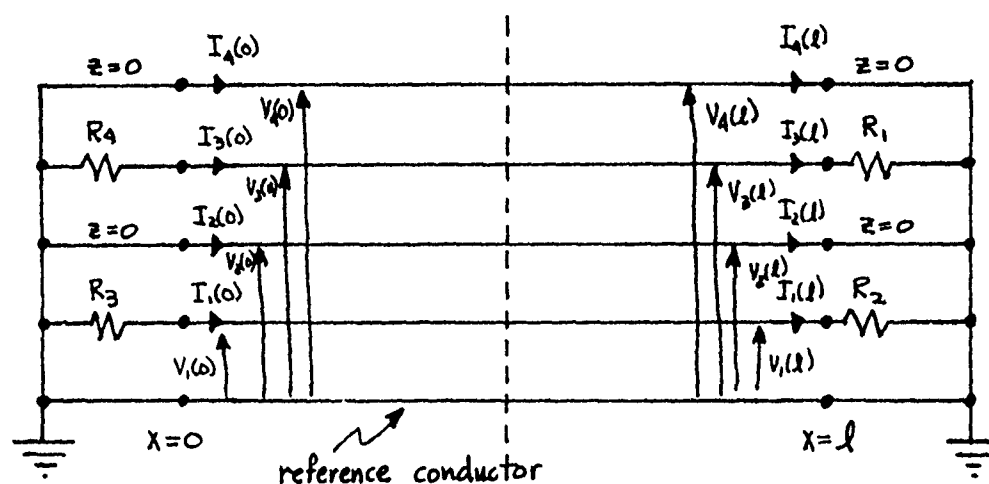


Figure 5.2 Five-wire, ground-signal-ground cable.

The equations defining the voltages at the ends of the lines are [2]

$$\begin{aligned}\underline{V}(0) &= \underline{E}_0 - \underline{Z}(0)\underline{I}(0) \\ \underline{V}(l) &= \underline{E}_l + \underline{Z}(l)\underline{I}(l)\end{aligned}\quad (5.29)$$

where \underline{E}_0 and \underline{E}_l are $(n-1) \times 1$ vectors of the equivalent open circuit port excitations, and $\underline{I}(0)$ and $\underline{I}(l)$ are $(n-1) \times 1$ vectors of current in the lines directed in the direction of increasing x . Writing the terminal impedance matrices, $\underline{Z}(0)$ and $\underline{Z}(l)$, at $x=0$ and $x=l$ respectively, for the 5 wire cable

in Fig. 5.2 yields [2, 13]:

$$\begin{aligned}\tilde{z}(0) &= \begin{bmatrix} R_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \tilde{z}(1) &= \begin{bmatrix} R_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} .\end{aligned}\quad (5.30)$$

The transmission line equations can then be solved in the normal fashion as described in [2, 13, 19].

CHAPTER VI

DESCRIPTION OF THE COMPUTER PROGRAM

VI.1 Introduction

The main purpose of this work was to implement on a digital computer the techniques of Chapter II to solve for the per-unit-length generalized capacitance matrix for a multi-conductor ribbon cable. Then, utilizing the results of Chapter V, to solve for the per-unit-length transmission line capacitance matrix needed for the analysis of crosstalk in ribbon cable systems. A listing of the program which will be described is contained in Appendix B.

VI.2 Computer Program GETCAP

GETCAP (which is an acronym for Generalized and Transmission line Capacitance matrices) is a FORTRAN computer program which utilizes all the results of this report to find the per-unit-length generalized and transmission line capacitance matrices for ribbon cables. The GETCAP main computer program uses a subroutine which is also called GETCAP. This GETCAP subroutine may be used alone in cases where it will be part of a larger program designed for specific purposes, or with its controlling mainprogram which provides a very user-oriented method of determining the capacitance matrices.

The problem to be solved is described by the following input variables:

- NW - The number of wires in the cable.
- RC - The radii of the conductors.
- RD - The radii of the outer dielectric insulation surfaces.
- CCSEP - The center-to-center separation of any two adjacent conductors.
- ER - The relative dielectric constant of the insulation material (relative to free space).
- NFC - The number of Fourier series terms to be used to represent the charge distributions around the conductor surfaces. (Note: The charge distribution is assumed to be of the form

(NFC-1)

$$\sigma_i(\theta) = \sum_{m=0} \sigma_{im} \cos m\theta$$

for $i=1, \dots, NW$.)

- NFD - The number of Fourier series terms to be used to represent the charge distributions around the dielectric surfaces. (Note: The charge distribution is assumed to be of the form

(NFD-1)

$$\sigma'_i(\theta) = \sum_{m=0} \sigma'_{im} \cos m\theta$$

for $i=1, \dots, NW$.)

- NF - The total number of Fourier coefficients per wire; ($NF = NFC + NFD$).
- IREF - The reference conductor for the transmission

line voltages; ($1 \leq \text{IREF} \leq \text{NW}$).

IOPT - A program option selector.

The cable dimensions, RC, RD, and CCSEP can appear in any choice of units (mm., meters, mils., etc.), so long as all of the dimensions are in the same units.

Subroutine GETCAP recognizes three possible values for IOPT.

- IOPT = 0 - Matrix partitioning is used to invert the \tilde{D} matrix.
- IOPT = 1 - Standard full inversion of the \tilde{D} matrix.
- IOPT = 2 - The dielectrics are removed. The program assumes a bare wire cable.

In addition, the GETCAP mainprogram recognizes options 10, 11, and 12. The second digit corresponds to options 0, 1, and 2 above. The first digit, 1, provides a copy of the upper triangle of the transmission line capacitance matrix in punched card format being generated for later use.

At this point, a discussion of the partitioning method which is selected with IOPT = 0 (or 10) is necessary. As was shown in Chapter III, the generalized capacitance matrix for ribbon cables is ideally symmetric with respect to both the main diagonal and the cross-diagonal. This means that only a few terms of the entire matrix must be found, as shown in Eq. (3.8). Further investigation shows that these terms can be developed from the elements of half of the inverse of the \tilde{D} matrix.

The elements of \tilde{C} are developed from the inverse of the

\tilde{D} matrix according to Eq. (2.43)

$$\tilde{C}_{ij} = 2\pi r_c \left[\sum_{m=1}^n T_{ij}^{1m} + r_d \sum_{m=1}^n T_{i'j}^{1m} \right] \quad (6.1)$$

where $\tilde{T} = \tilde{D}^{-1}$. The symmetry of \tilde{C} with respect to both the main and cross-diagonals will allow those elements which completely specify \tilde{C} to be developed only once. Also, these elements will be chosen so that only the submatrices on the left half of \tilde{T} will be used.

Noble [12] describes a technique for finding the inverse of a square, nonsingular matrix \tilde{D} partitioned such that

$$\tilde{D} = \begin{bmatrix} \tilde{P} & \tilde{Q} \\ \tilde{R} & \tilde{S} \end{bmatrix} \quad (6.2)$$

where \tilde{P} and \tilde{S} are square matrices. The inverse of \tilde{D} (if \tilde{D} and \tilde{S} are non-singular) is

$$\tilde{D}^{-1} = \begin{bmatrix} \tilde{X} & -\tilde{X}\tilde{Q}\tilde{S}^{-1} \\ -\tilde{S}^{-1}\tilde{R}\tilde{X} & \tilde{W} \end{bmatrix}$$

$$\text{where } \tilde{X} = (\tilde{P} - \tilde{Q}\tilde{S}^{-1}\tilde{R})^{-1} \text{ and } \tilde{W} = \tilde{S}^{-1} + \tilde{S}^{-1}\tilde{R}\tilde{X}\tilde{Q}\tilde{S}^{-1} \quad (6.3)$$

Partitioning the matrix \tilde{D} of Eq. (6.2) in halves results in the minimal amount of time required to solve for part of the inverse of \tilde{D} by the method in (6.3). This can be verified by counting the number of operations (multiplications and divisions) required to obtain the form of \tilde{D}^{-1} in Eq. (6.3). The number of operations required to multiply an $n \times k$ matrix

by a $k \times p$ matrix is $n \times p$.

Partitioning \tilde{D} in Eq. (6.2) in halves, the number of operations to solve for the required portions of \tilde{D}^{-1} as described above is approximately $5/8$ of the number of operations required by a direct full inversion of \tilde{D} . To show this, consider \tilde{D} of order $2n$ such that

$$\tilde{D} = \begin{bmatrix} \tilde{P} & \tilde{Q} \\ \tilde{R} & \tilde{S} \end{bmatrix}, \quad (6.4)$$

where \tilde{P} , \tilde{Q} , \tilde{R} , and \tilde{S} are $n \times n$ matrices. To solve for \tilde{D}^{-1} directly will result in approximately $8n^3$ operations. If the inverse of \tilde{D} is written as

$$\tilde{D}^{-1} = \begin{bmatrix} \tilde{\alpha} & \tilde{\beta} \\ \tilde{\gamma} & \tilde{\delta} \end{bmatrix}, \quad (6.5)$$

then solving for $\tilde{\alpha}$ and $\tilde{\gamma}$ will require $5n^3$ operations (remembering that n^3 operations are required to invert an $n \times n$ matrix and n^3 operations are also required to multiply two $n \times n$ matrices).

VI.3 Operational Details of the GETCAP Program

The entire GETCAP program consists of five program units:

- MAIN - The main program for inputting data and controlling output of results.
- GETCAP - The subroutine which performs the actual computation of the capacitance matrices from the input data.

- MINV - A matrix inversion subroutine from the IBM Scientific Subroutine Package (SSP) [16].
- MPC - A subroutine which multiplies two general matrices, then multiplies the resulting matrix by a constant.
- MPRT - A subroutine which outputs a general matrix to the printer in matrix format with labeling.

Each of these program units will be described separately in this section, with the greatest emphasis on the GETCAP subroutine.

The main program was written so as to be used by persons who are not computer-specialists. Its operational characteristics are illustrated by the flowchart in Fig. 6.1. The main program is an executive over the GETCAP subroutine; it also checks the input data for obvious errors. The main program also provides the matrix and array storage areas used in the computations by subroutine GETCAP. The user must ensure that the array dimensions are large enough to deal with the problems to be considered. Directions for this dimensioning are given at the beginning of the listing of the main program. Assuming a maximum number of wires, NW, and a maximum number of total Fourier series expansion terms per wire, $NF=(NFC+NFD)$, for all problems to be considered, the dimensions of the matrices required in subroutine GETCAP will be as follows:

$$C(NW^2)$$

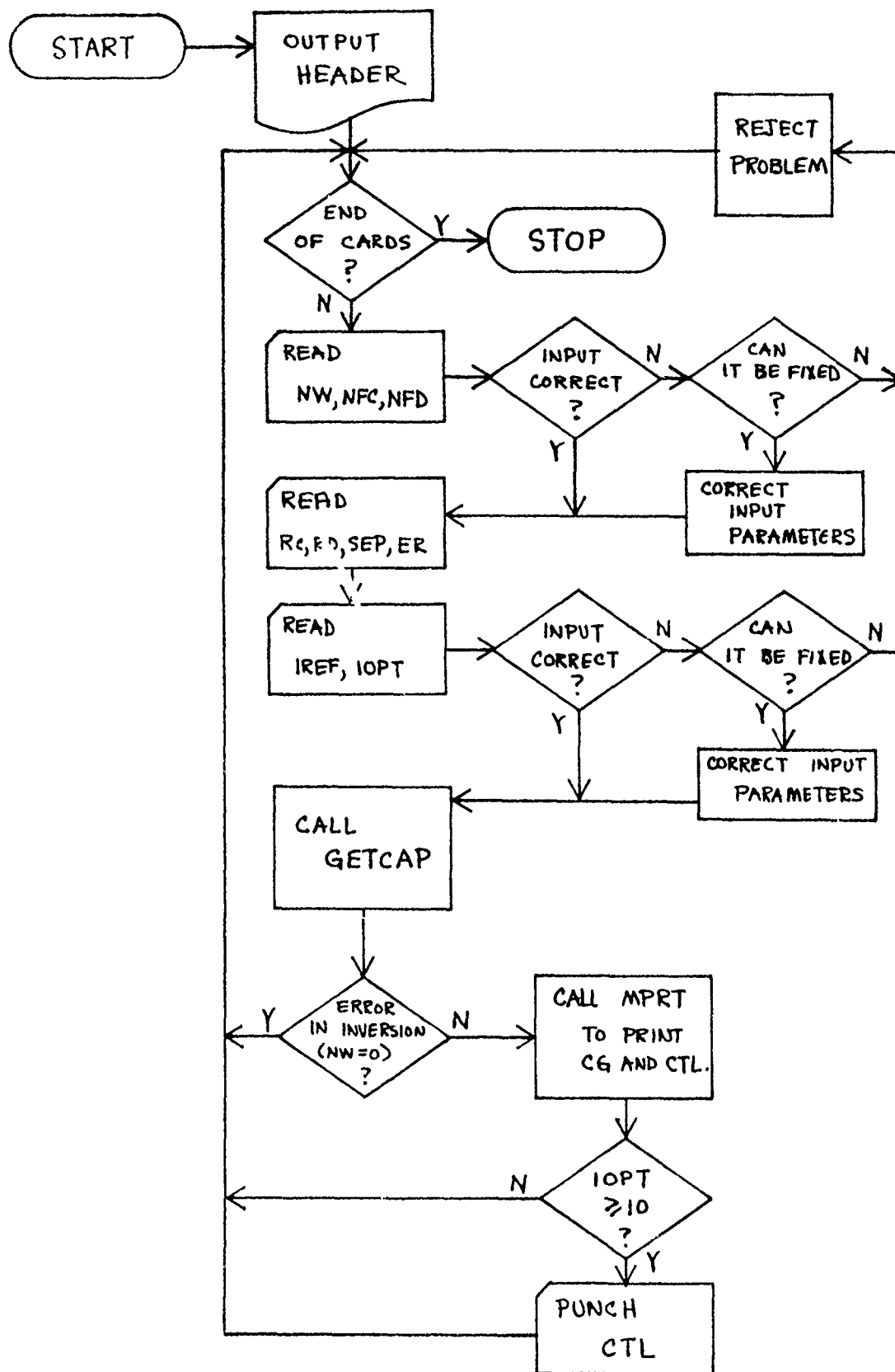


Figure 6.1. Flowchart of MAIN program.

$$\begin{aligned}
&C1((NW-1)^2) \\
&D(NF^2) \\
&D1((NW\ NF)^2) \\
<(2\ NF\ NW) \\
&SCR((NF\ NW + 1)/2) \ .
\end{aligned}$$

Note that these items are stored in vector format. This dimensioning will allow all problems up to the maximum specified to be considered. When the GETCAP subroutine returns to the main program, a printed output of the generalized and transmission line capacitance matrices is generated by calling subroutine MPRT.

Subroutine GETCAP is the computational part of the GETCAP program. All input data, output data, and working vectors are passed through the argument list as shown below.

```

SUBROUTINE GETCAP (NW,NFC,NFD,NF,RC,RD,CCSEP,ER,IREF,
                  IOPT,CG,CTL,D,D1,SCR,LT)

```

The first ten arguments are the input variables described earlier. CG and CTL are the resulting per-unit-length generalized and transmission-line capacitance matrices, respectively. The last four arguments are working vectors used by GETCAP. D1 is matrix \tilde{D} in Eq. (2.28) stored in vector format.

Subroutine GETCAP operates as shown in the flowchart of Fig. 6.2. The first step is a scaling of the input wire dimensions to prevent numerical underflow and overflow by the computer in generating the inverse of the D1 matrix. Equations (2.17) and (2.18) are then used to compute the

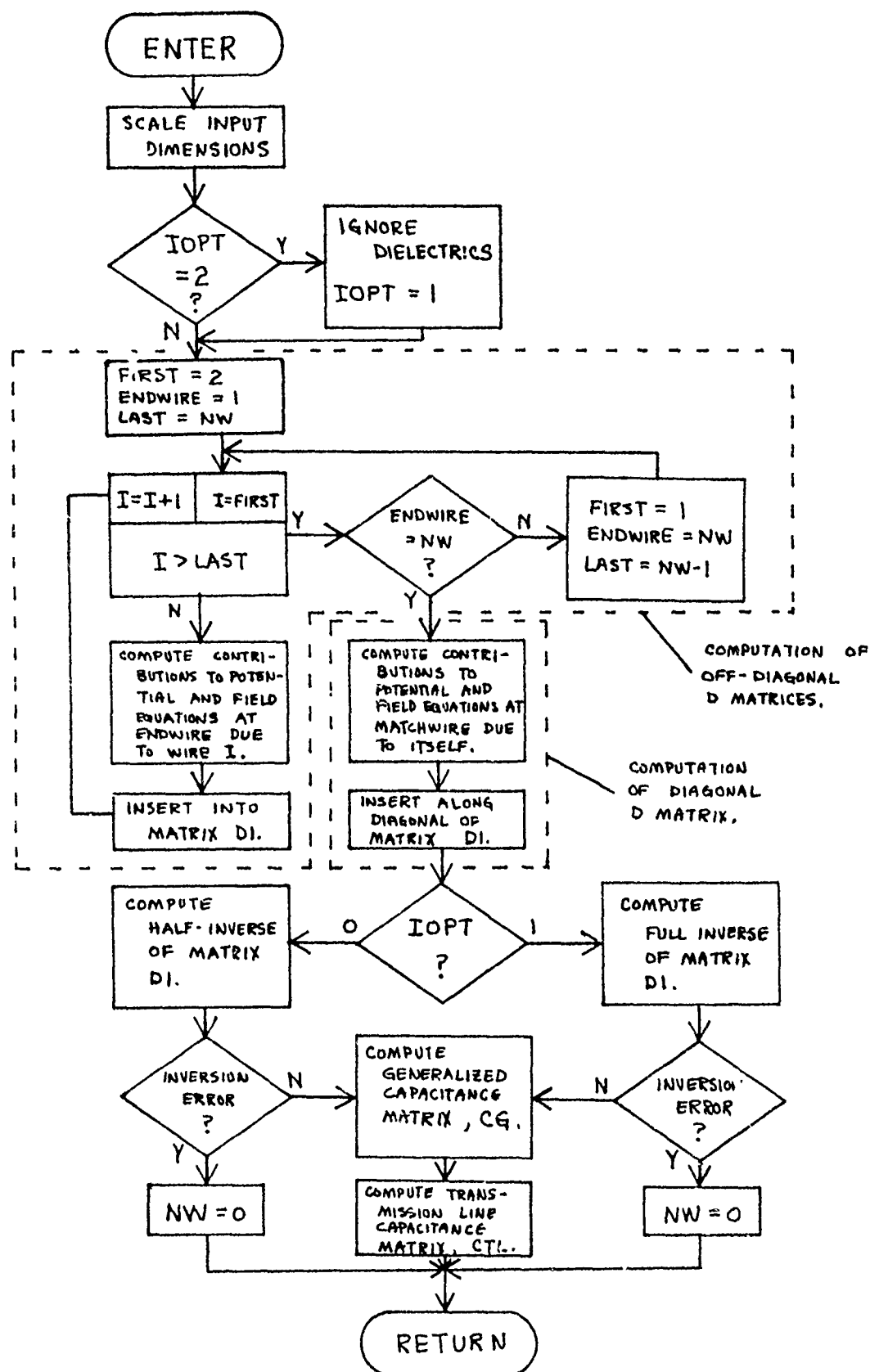


Figure 6.2. Basic Flowchart of Subroutine GETCAP.

off-diagonal submatrices \underline{D}_n , for $n = \pm 1, \dots, \pm(n-1)$, shown in Eq. (2.28). These submatrices are then inserted into the large matrix D1 according to the relationship of Eq. (2.28).

Equations (2.17), (2.18), (2.19), and (2.20) are then used to generate the diagonal block submatrix \underline{D}_0 of Eq. (2.28). \underline{D}_0 is then inserted in the diagonal blocks of the D1 matrix. Computer matrix D is used to hold each of the block submatrices \underline{D}_n , $n=0, \pm 1, \dots, \pm(n-1)$ for insertion into D1.

It should be noted that when IOPT = 0 (or 10), the \underline{D} matrix in Eq. (6.1) is stored in the computer in the vector D1, such that the first $(NF \times NW/2)^2$ locations correspond to matrix \underline{P} , the next $(NF \times NW/2)^2$ locations correspond to matrix \underline{Q} , followed by $(NF \times NW/2)^2$ locations for \underline{R} , and then $(NF \times NW/2)^2$ locations for \underline{S} . In this case, the submatrices in \underline{D} are inserted into the D1 matrix to take into account this unusual storage order.

When IOPT=2 (or 12), the dielectrics are ignored. Thus, equations (2.18) and (2.20) for the electric field vector at a point are not used in generating the \underline{D} submatrices. Also, since the dielectric surfaces are ignored, the contributions to the conductor potentials due to the charges on these surfaces are ignored.

Following the generation of matrix D1, it is inverted according to the method designated by IOPT. If IOPT=1 (or 11), subroutine MINV inverts the entire D1 matrix. If IOPT=0 (or 10), the method described by Eq. (6.3) is used to

compute half of the inverse of $D1$.

The matrix \tilde{D} represented by Eq. (6.2) is stored in $D1$ and inverted according to the following procedure:

- 1) Invert matrix S using MINV. S is replaced by its inverse \tilde{S} .
- 2) Multiply $\tilde{S}^{-1} R$, and insert the result in S using MPC.
- 3) Multiply Q by \tilde{S}^{-1} using MPC and insert the result $(Q \tilde{S}^{-1} \tilde{R})$ in Q .
- 4) Subtract $Q \tilde{S}^{-1} R$ from P and insert the result $(P - Q \tilde{S}^{-1} R)$ into P .
- 5) Invert $(P - Q \tilde{S}^{-1} R)$ using MINV. Store the result in P .
- 6) Multiply $\tilde{S}^{-1} R$ by $(P - Q \tilde{S}^{-1} R)^{-1}$ using MPC, then multiply the result by \tilde{S}^{-1} and store in R .

Half of the inverse of $D1$ then lies in partitions P and R . This method takes slightly over half the time required to invert the entire $D1$ matrix.

Once the $D1$ matrix is inverted, the elements of the generalized capacitance matrix, \tilde{C} , are computed from the elements of the inverse of $D1$ by Eq. (2.43) and stored in the computer as matrix CG . When $IOPT=1$ (or 11), all the elements of \tilde{C} are computed. For $IOPT=0$ (or 10), the symmetry conditions as described in Section III.3 are utilized and only a portion of the elements of \tilde{C} are computed, as indicated by Eq. (3.8). The rest of the elements are assumed to be duplicates of those computed.

The transmission line capacitance matrix, \tilde{C} , is then computed from the generalized capacitance matrix by following the algorithm in Section V.2, and the elements of \tilde{C}

are stored in the computer as matrix CTL. Subroutine GETCAP then returns to the main program with both the per-unit-length generalized capacitance matrix, CG, and the per-unit-length transmission line capacitance matrix, CTL.

Subroutine MINV is a standard matrix inversion routine included in the IBM Scientific Subroutine Package [16]. Its argument list is shown below:

SUBROUTINE MINV(A, N, D, L, M)

The arguments have the following meaning:

- A - An input matrix which is destroyed in computation and replaced by the inverse of A.
- N - The order of matrix A.
- D - The resultant determinant of A.
- L - A working vector of length N.
- M - A working vector of length N.

Subroutine MPC is used to find the product of two general matrices and to multiply the resulting matrix product by a constant. The argument list for MPC is shown below.

SUBROUTINE MPC (A, B, R, S, L, M, N, C)

These arguments have the following significance:

- A - The first matrix, dimensioned L by M.
- B - The second matrix, dimensioned M by N.
- R - The resulting matrix, dimensioned L by N.
- S - A scratch vector, length L.
- L - The number of rows in A and R.
- M - The number of columns in A (number of rows in B).

N - The number of columns in B and R.

C - The constant by which R is multiplied.

The multiplication takes the form:

$$\underline{R} = \underline{C} \underline{A} \underline{B} \quad .$$

The subroutine was written so that neither A nor B are destroyed by the computation. In addition, if the dimensions of matrix R are large enough (N greater than or equal to M), the use of scratch vector S provides that A and R can share the same storage locations. This is useful when two large matrices are to be multiplied, and the first one is no longer needed after the multiplication, thus saving the amount of storage needed by the matrices. In the GETCAP program, the large size of the D1 matrix makes this savings considerable. The flowchart of Fig. 6.3 illustrates operation of the MPC subroutine.

Subroutine MPRT is used for printing a matrix in matrix format. Elements are printed in scientific notation rounded to three digits. The rows and columns are numbered on the printout to provide easy identification of any element.

The argument list of MPRT is shown below:

SUBROUTINE MPRT (A, M, N, B, J)

where the arguments have the following meaning:

A - The matrix to be printed.

M - The number of rows in A.

N - The number of columns in A.

B - The title of the matrix in Hollerith format.

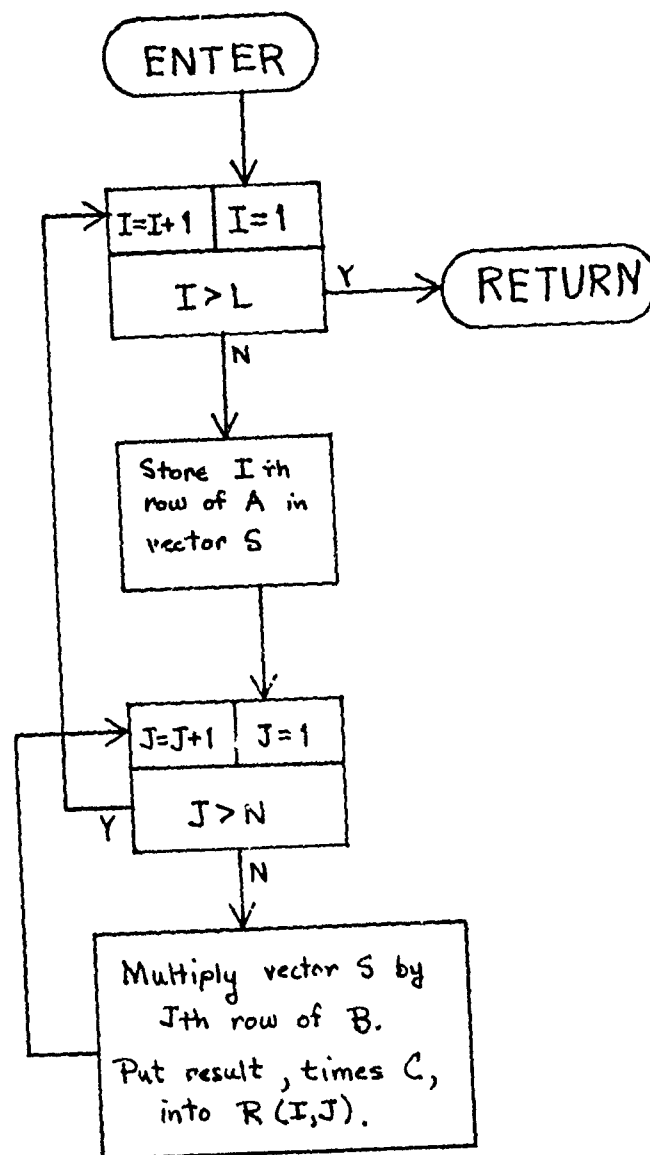


Figure 6.3. Flowchart of Subroutine MPC.

J - The number of characters in the title.

The title, given in B, is printed on the header of the first page of the printout. The first twelve columns are then printed on page two, and so forth, until the entire matrix has been printed. The printout can then be separated, and page two connected to the right of page one, page three to the right of page two, etc., so that the printed matrix can be viewed in its normal matrix form. An example of a general matrix printed with MPRT is given in Fig. 6.4. The calling statement was

```
CALL MPRT(A,6,7,'TYPICAL OUTPUT OF A',19)
```

with the contents of A stored as shown in the printout.

The listing of the program found in Appendix B includes comment statements to provide an interested user the ability to check or find any particular portion of the program. The program was designed to operate in as little time as possible, using as little duplicated storage as possible. The object code consumes approximately 167K bytes for the capability to work with up to twenty wires and a total of ten Fourier series terms per wire. Run times can be significantly reduced by running the program on the FORTRAN G or H compilers with an object deck. Compilation is no longer necessary, and execution time is much faster than on diagnostic compilers such as WATFIV available at the University of Kentucky. Typical run times on the IBM System 370/165 at the University of Kentucky using the FORTRAN G compiler with no printout of the source listing are shown below.

TYPICAL OUTPUT OF A MATRIX - - PAGE 1

	COLUMN						
	1	2	3	4	5	6	7
ROW	*****						
1	1.00D 00	2.00D 00	3.00D 00	4.00D 00	5.00D 00	6.00D 00	7.00D 00
2	8.00D 00	9.00D 00	1.00D 01	1.10D 01	1.20D 01	1.30D 01	1.40D 01
3	1.50D 01	1.60D 01	1.70D 01	1.80D 01	1.90D 01	2.00D 01	2.10D 01
4	2.20D 01	2.30D 01	2.40D 01	2.50D 01	2.60D 01	2.70D 01	2.80D 01
5	2.90D 01	3.00D 01	3.10D 01	3.20D 01	3.30D 01	3.40D 01	3.50D 01
6	3.60D 01	3.70D 01	3.80D 01	3.90D 01	4.00D 01	4.10D 01	4.20D 01

Figure 6.4 Sample Output Generated by Subroutine MPRT.

COMPILE TIME	- 1.9 sec.
EXECUTION TIME *	- 37.5 sec.

VI.4 Verification of Program Operation

Unfortunately, there is no way of comparing these computed results to known data on the capacitance matrices of ribbon cables since none apparently exists. Comparisons can be made, however, to known exact and approximate formulas for the capacitance where the dielectric effect is ignored; i.e., bare wires.

The data in Table 8 show how the capacitance for the two bare-wire case converges rapidly to the exact value found by the equation [5]

$$C_0 = \frac{\pi \epsilon_v}{\cosh^{-1}\left(\frac{d}{2r_c}\right)} \quad , \quad (6.7)$$

where d is the center-to-center separation of the wires, and r_c is the radius of the conductor. The per-unit-length inductance L is related to C_0 by Eq. (5.16). Ribbon cables typically have a conductor separation greater than four times the conductor radius. In this range, results for the per-unit-length inductance for a two-wire cable are extremely accurate with even small numbers of Fourier series terms.

Results from GETCAP for a multiwire bare cable assembly can be compared to the approximate transmission line capacitance matrix derived in Chapter V. These results are shown

* 20 wires, 10 coefficients per wire, IOPT=0.

Separation:	2.1	2.5	3.0
Exact:	88.32663366390219	40.13035670976953	28.90228602008718
Program:			
NF = 1	32.95277186987344	28.08301159327916	24.16088221267191
2	53.76411761858517	36.91825147471941	28.24450644566931
3	67.83411379795934	39.32969351890125	28.81260103496420
4	76.66772365824638	39.93397813126190	28.88987614313759
5	81.86785891927163	40.08222038915016	28.90054582896143
6	84.80464551776120	40.11852864232260	28.90203971793245
7	86.42327830561355	40.12744302360560	28.90225093176778
8	87.30320685998552	40.12963748183029	28.90228099789533
9	87.77790711487611	40.13017888027915	28.90228529875102
10	88.03290618021939	40.13031268305180	28.90228591620657

Separation:	4.0	10.0
Exact:	21.12158913598862	12.13394666352968
Program:		
NF = 1	19.63582704888583	12.054395332335852
2	21.03180468263799	12.13334690616347
3	21.11582282108362	12.13394129671470
4	21.12120303931120	12.13394661256600
5	21.12156267957364	12.13394666303271
6	21.12158729834855	12.13394666352477
7	21.12158900725528	12.13394666352964
8	21.12158912691928	12.13394666352969
9	21.12158913534718	12.13394666352969
10	21.12158913594313	12.13394666352969

Conductor radius,
r_c = 1.0

All elements *10⁻¹², pF/M

Table 8 Comparison of exact and results from GETCAP for the transmission--
line capacitance of a two-wire cable with no dielectric.

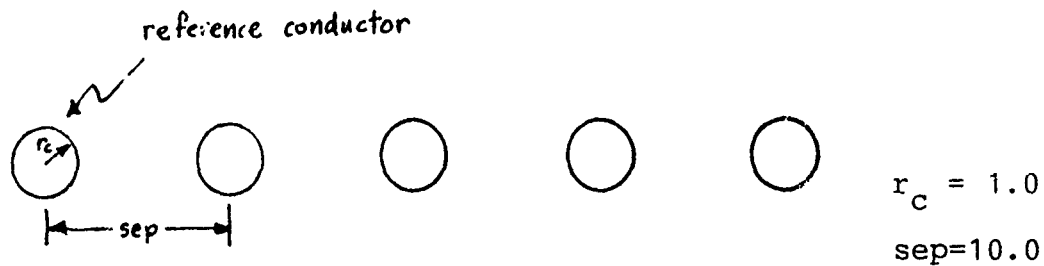


Figure 6.5 Five wire bare ribbon cable.

in Table 9 for the cable shown in Fig. 6.5. The approximate results agree with the GETCAP results to at least two digits, and the largest error is approximately 2% between the two sets of results.

Another check was made of the results from GETCAP by checking the rate of convergence of the elements in the first row of the generalized capacitance matrix for a five wire ribbon cable (with the dielectric included) as shown in

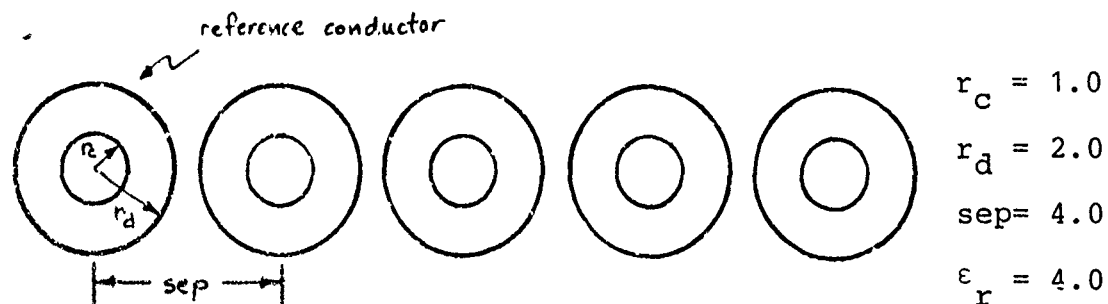


Figure 6.6 Five wire flatpack cable.

Fig. 6.6. The results in Fig. 6.7 indicate smooth convergence to the final values for increasing values of NF. (The magnitudes of the elements are plotted since the off-diagonal terms of \tilde{C} are negative.) These results are for the worst case problem (dielectric boundaries touching, high relative dielectric constant).

Elements of the Transmission-Line Capacitance
Matrix for a Five-wire Bare Cable

(Elements $\times 10^{-12}$, pF/M)

Element	GETCAP Results	Approximate
C(1,1)	18.87646053717670	18.78148468923442
C(1,2)	-6.851495768047740	-6.854631737467468
C(1,3)	-2.129410716129314	-2.066002017837609
C(1,4)	-1.843114316184374	-1.825053886227327
C(2,2)	19.14682911214455	19.03688696351830
C(2,3)	-6.851495768052782	-6.854631737467466
C(2,4)	-2.721918788019494	-2.663811744291685
C(3,3)	18.87646053717669	18.78148468923442
C(3,4)	-8.052439732815269	-8.034797047702019
C(4,4)	14.81610887311056	14.72743429621767

Physical conditions: $r_c = 1.0$
sep = 10.0

End wire chosen as reference.

Table 9 Comparison of GETCAP results to the approximate formula derived in Chapter V for the capacitance matrix of multi-conductor bare wire.

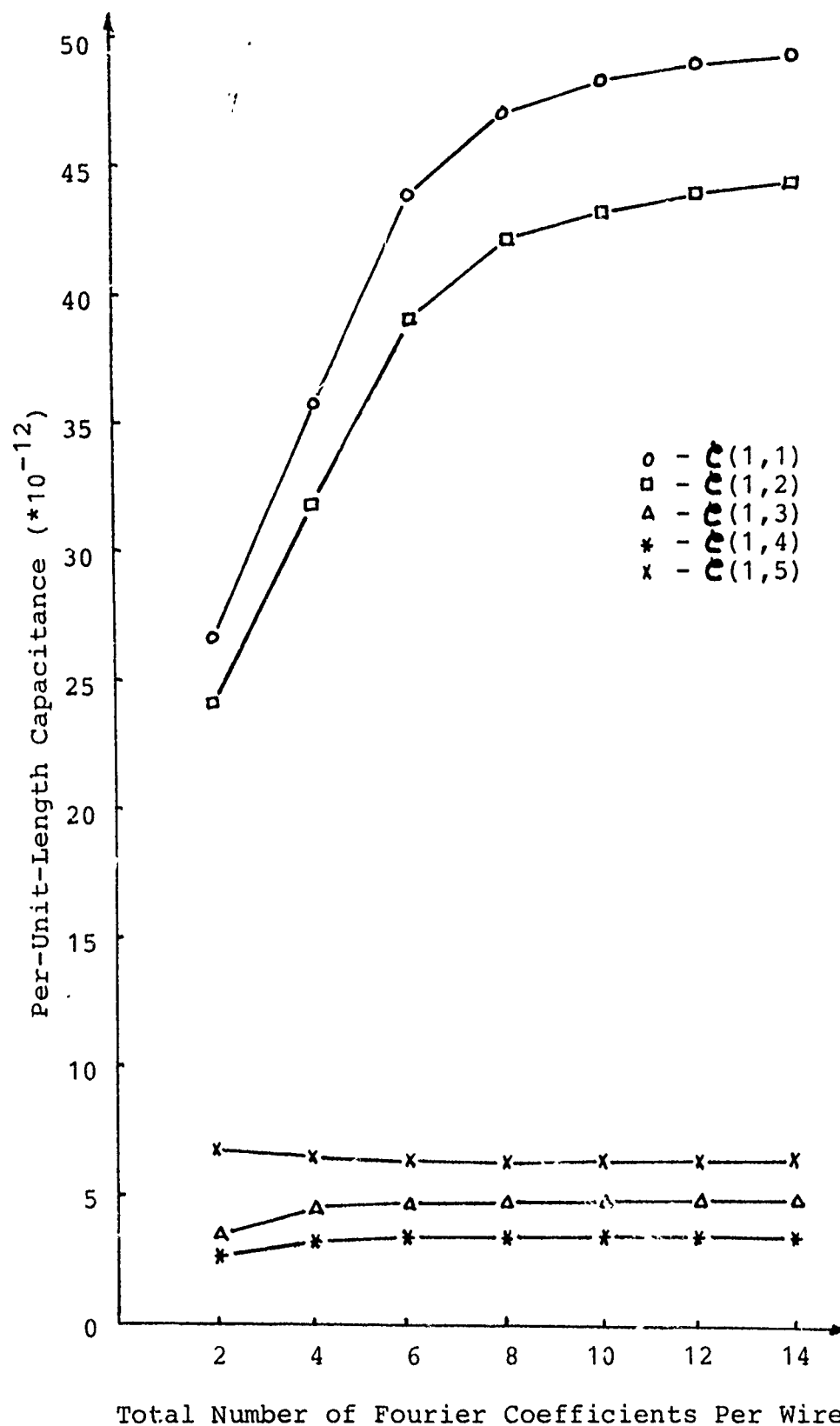


Figure 6.7. Convergence of the elements of the generalized capacitance matrix.

CHAPTER VII

USING GETCAP

VII.1 Introduction

This chapter will describe the use of the GETCAP program for two different purposes. Section VII.2 will consider use of the GETCAP subroutine for cases where it will be required to compute and deliver the generalized and transmission line capacitance matrices to other program units written by the user. More commonly, though, the transmission line capacitance matrix of the ribbon cable is used as input data for another program, such as in a cross-talk analysis. Section VII.3 will describe the entire GETCAP program with its facilities for outputting the elements of the transmission line capacitance matrix in punched card format.

VII.2 Using Subroutine GETCAP

Subroutine GETCAP is the workhorse of the GETCAP program. It is used by calling the subprogram from another FORTRAN program unit, using the following statement and argument list:

```
CALL GETCAP (NW, NFC, NFD, NF, RC, RD, CCSEP, ER,  
             IREF, IOPT, CG, CTL, D, D1, SCR, LT)
```

As described in Chapter VI, the arguments have the following

meaning:

- NW - Number of wires in the cable.
- NFC - Number of Fourier series terms to be used to represent the charge distributions around the dielectric surfaces. Note: the charge distribution is assumed to be of the form

$$\sigma_i(\theta) = \sum_{m=0}^{(NFC-1)} \sigma_{im} \cos m\theta \quad \text{for } i=1, \dots, NW.$$

- NFD - Number of Fourier series terms to be used to represent the charge distributions around the dielectric surfaces. Note: the charge distribution is assumed to be of the form

$$\sigma'_i(\theta) = \sum_{m=0}^{(NFD-1)} \sigma'_{im} \cos m\theta \quad \text{for } i=1, \dots, NW.$$

(See Section III.3 for a discussion of how to choose these.)

- NF - Total number of Fourier series terms per wire. (NF = NFC + NFD).
- RC - Radii of the conductors.
- RD - Radii of the outer dielectric insulation surfaces.
- CCSEP - Center-to-center separation of any two adjacent conductors.

(These cable dimensions must appear in the same units: meters, mils, inches, etc.)

- ER - Relative dielectric constant of the insulation material (relative to free space).
- IREF - The reference conductor for the transmission line voltages. ($1 \leq \text{IREF} \leq \text{NW}$)
- IOPT - Option selector.
0 - fast solution
1 - long matrix inversion
2 - dielectric insulations removed (bare wire case)
- CG - Holds the computed per-unit-length generalized capacitance matrix in vector format. (dimension NW^2)

- CTL - Holds the computed per-unit-length transmission line capacitance matrix in vector format.
(dimension $(NW-1)^2$)
- D - A working matrix of dimension $NF \times NF$.
- D1 - A working vector of length $(NF \cdot NW)^2$.
- SCR - A scratch vector of length $\frac{NF(NW+1)}{2}$.
- LT - A scratch vector of length $2 \cdot NF \cdot NW$.

All that is required in using subroutine GETCAP is to provide for and properly dimension these required matrices and vectors in the calling program. One must be especially careful about the size of vector D1. A typical case of twenty wires and ten Fourier coefficients per wire requires that D1 be dimensioned to provide $(20 \times 10)^2 = 40,000$ storage locations. The user should note that D1 is dimensioned in the main program as a matrix $D1(NF \cdot NW, NF \cdot NW)$. In subroutine GETCAP, however, D1 is stored in vector format.

GETCAP requires two other subroutines. They are:

- MINV - A standard IBM SSP matrix inversion program [16]
- and MPC - The product of two matrices with a scalar constant multiplying the result.

Both of these routines are included in the program listing found in Appendix B.

VII.3 Using the GETCAP Program

The full GETCAP program is a user-oriented method to find the capacitance matrices of ribbon cables. The input to

[illegible]

1

1

The physical characteristics card would be:

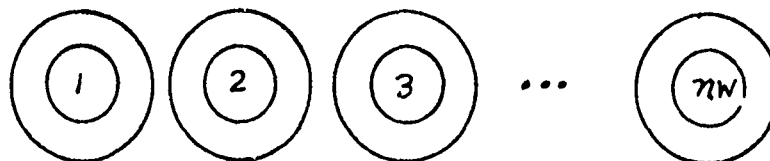
Col. 1	Col. 11	Col. 21	Col. 31
└───┘	└───┘	└───┘	└───┘
.16002	.508	1.27	3.5

(3) Option card:

Cols. 1-3, IREF -- the reference conductor.

Note:

Conductor numbering is sequential from the end of the cable, as illustrated below:



Therefore, IREF can range from 1 to NW.

Cols. 4-6, IOPT -- option.

IOPT = 0	fast solution
IOPT = 1	long inversion
IOPT = 2	dielectric insulation removed
IOPT = 10	} same as 0, 1, and 2 above except that the upper triangle of the transmission-line capacitance matrix is punched on cards.
IOPT = 11	
IOPT = 12	

Example:

Assume conductor 1 chosen as reference and execute in fastest time, with a punched copy of the transmission line capacitance matrix generated.

Then IOPT = 10 and the option card is:

Col. 1	Col. 4
└───┘	└───┘
.1	10

Typical output from the GETCAP program is shown in Fig.

7.2. Fig. 7.2a shows the header page which is printed at the beginning of a batch of problems to the program. Fig. 7.2b is the first page of output associated with a problem.

GGG	EEEE	TTTTT	CCC	AAA	PPPP
G G E		T	C C	A A	P P
G E		T	C	A A	P P
G GG EEEE		T	C	AAAAA	PPPP
G G E		T	C	A A	P
G G E		T	C C	A A	P
GGGG EEEEE		T	CCC	A A	P

GGG	EEEE	TTTTT	CCC	AAA	PPPP
G G E		T	C C	A A	P P
G E		T	C	A A	P P
G GG EEEE		T	C	AAAAA	PPPP
G G E		T	C	A A	P
G G E		T	C C	A A	P
GGGG EEEEE		T	CCC	A A	P

GENERALIZED AND TRANSMISSION LINE CAPACITANCE MATRICES
OF RIBBON CABLES

B A T C H R U N

Fig. 7.2a Typical Output from the GETCAP program.

```

*****
GENERALIZED AND TRANSMISSION LINE CAPACITANCE MATRICES OF "FLAT-PACK" CABLE

CONDITIONS:      3 FOURIER COEFFICIENTS AROUND THE CONDUCTOR, AND  7 AROUND THE DIELECTRIC

PHYSICAL CHARACTERISTICS:
CONDUCTOR RADIUS= 1.6002E-04 (METERS)
DIELECTRIC RADIUS= 5.0800E-04 (METERS)
CENTER-TO-CENTER SEPARATION= 1.2700E-03 (METERS)
RELATIVE DIELECTRIC CONSTANT= 3.500E 00

REFERENCE CONDUCTOR IS NO.  1
OPTION NUMBER IS  0
*****

```

Fig. 7.2b Typical output from the GETCAP program.

GENERALIZED CAPACITANCE MATRIX - - PAGE 1

ROW	COLUMN	1	2	3	4	5
1	*	1.85E-11	-2.35E-11	-7.31E-12	-6.43E-12	-1.20E-11
2	*	-2.35E-11	3.88E-11	-1.90E-11	-4.54E-12	-6.43E-12
3	*	-7.31E-12	-1.90E-11	3.95E-11	-1.90E-11	-7.31E-12
4	*	-6.43E-12	-4.45E-12	-1.90E-11	3.88E-11	-2.35E-11
5	*	-1.20E-11	-6.43E-12	-7.31E-12	-2.35E-11	1.85E-11

Fig. 7.2c Typical output from the GETCAP program.

TRANSMISSION LINE CAPACITANCE MATRIX - - PAGE 1

ROW	COLUMN	1	2	3	4
1	*	4.08E-11	-1.71E-11	-2.46E-12	-2.07E-12
2	*	-1.71E-11	4.11E-11	-1.71E-11	-3.41E-12
3	*	-2.46E-12	-1.71E-11	4.08E-11	-1.92E-11
4	*	-2.07E-12	-3.41E-12	-1.92E-11	2.76E-11

Fig. 7.2d Typical output from the GETCAP program.

All of the input data are printed as they are read in, and any errors in the data are flagged on this page. The item, (METERS), on this printout is used to show that all wire dimensions must be in the same units (meters, inches, mils, etc.). Fig. 7.2c is the output of the generalized capacitance matrix associated with the ribbon cable described by the data in Fig. 7.2b. The elements have units of Farads per meter. Fig. 7.2d, the transmission line capacitance matrix with a conductor chosen as reference for the line voltages, is the final page of output for each problem.

VII.4 Format of the Output of the Transmission Line Capacitance Matrix

An algorithm was developed in Chapter V which is used in the computer program to determine the transmission line capacitance matrix from the generalized capacitance matrix; any of the conductors in the ribbon cable can be chosen as the reference conductor in this algorithm.

In the printout of the transmission line capacitance matrix, the row and column indices are numbered from 1 to (NW-1). Thus, if the last wire, NW, is chosen as the reference, the indices will correspond to the elements correctly. However, if another conductor is chosen, the user will have the responsibility for correctly interpreting the results. For example, the case of a 10 wire cable can be considered. If conductor 5 is chosen as reference, $CTL_{5,1}$ is actually the element of C describing the capacitance between wires 6 and 1. Similarly, $CTL_{5,5}$ describes

the self capacitance of wire 6.

The punched card output is of a similar format. One upper triangular element is punched per card, with the row and column indices of the element punched as well. Suppose the capacitance matrices were determined for a twenty wire ribbon cable with the 20-th wire chosen as reference. A sample punched card would look as follows:

C 10 3 -4.384692E-10

This element would be $C_{10,3}$ with value $-4.384692 \times 10^{-10}$. The user has the responsibility for correctly interpreting the results if any wire other than the last wire is chosen as the reference conductor.

CHAPTER VIII

SUMMARY

A method for computing the capacitance matrix for dielectric-coated conductors was applied to the case of ribbon cables. Simplifications in the method were made possible by the symmetry of the cable dimensions; the radii of the conductors are all identical, the radii of the dielectric insulations are all identical, and the center-to-center spacing of adjacent wires is identical. In addition, the wires are oriented in a horizontal plane which is maintained throughout the length of the cable. An attempt was made to optimize the selection of matchpoints to ensure valid results and reduce computation time.

Using the generalized capacitance matrix, a technique for obtaining the transmission line capacitance and inductance matrices was developed; these matrices are used in frequency response and crosstalk analyses of cable systems. An approximate method for determining the transmission line inductance matrix was also developed.

Computer program GETCAP was written to utilize the results of this report. Given the wire dimensions and the number of wires in the cable, GETCAP will compute the per-unit-length generalized capacitance matrix, and from that

determine the per-unit-length transmission line capacitance matrix for a ribbon cable. Results indicate rapid convergence of the matrices to accurate values. The GETCAP program is written to be useful even to the person who is not a computer specialist, with simple input formats and error correcting facilities.

Ribbon cables are now widely used in the interconnection of electronic systems. The ability to compute the transmission line capacitance matrix for such cables enables the multiconductor transmission line equations to be solved. This in turn will enable a more precise analysis of ribbon cable systems through a detailed analysis of crosstalk.

APPENDIX A

The purpose of this appendix is to provide justification for omitting the reference potential terms in the potential expressions in Chapter II. Consider Fig. A.1(a) in which infinitesimal line charges lie on a cylindrical surface of radius r' . The potential $\phi(r, \theta)$ with respect to the potential reference point due to one of the line charges is (reference [7], pp. 91-92)

$$\phi(r, \theta) = - \frac{\lambda}{2\pi\epsilon_v} \ln\left(\frac{d}{d_0}\right) \quad (\text{A.1})$$

where the distances from the line charge to the potential and reference points are given by

$$d^2 = r^2 + r'^2 - 2rr'\cos(\theta' - \theta) \quad (\text{A.2a})$$

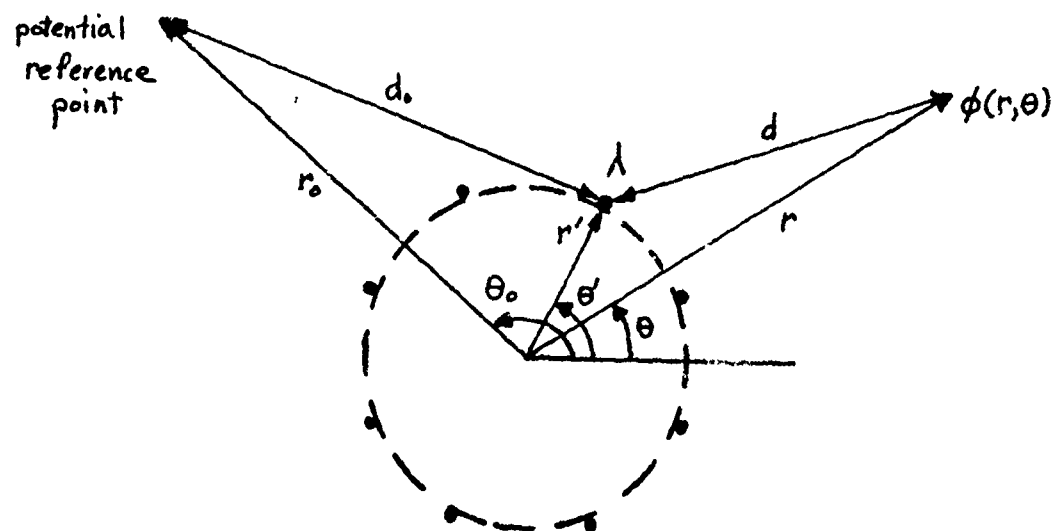
and

$$d_0^2 = r_0^2 + r'^2 - 2r_0r'\cos(\theta' - \theta_0) \quad (\text{A.2b})$$

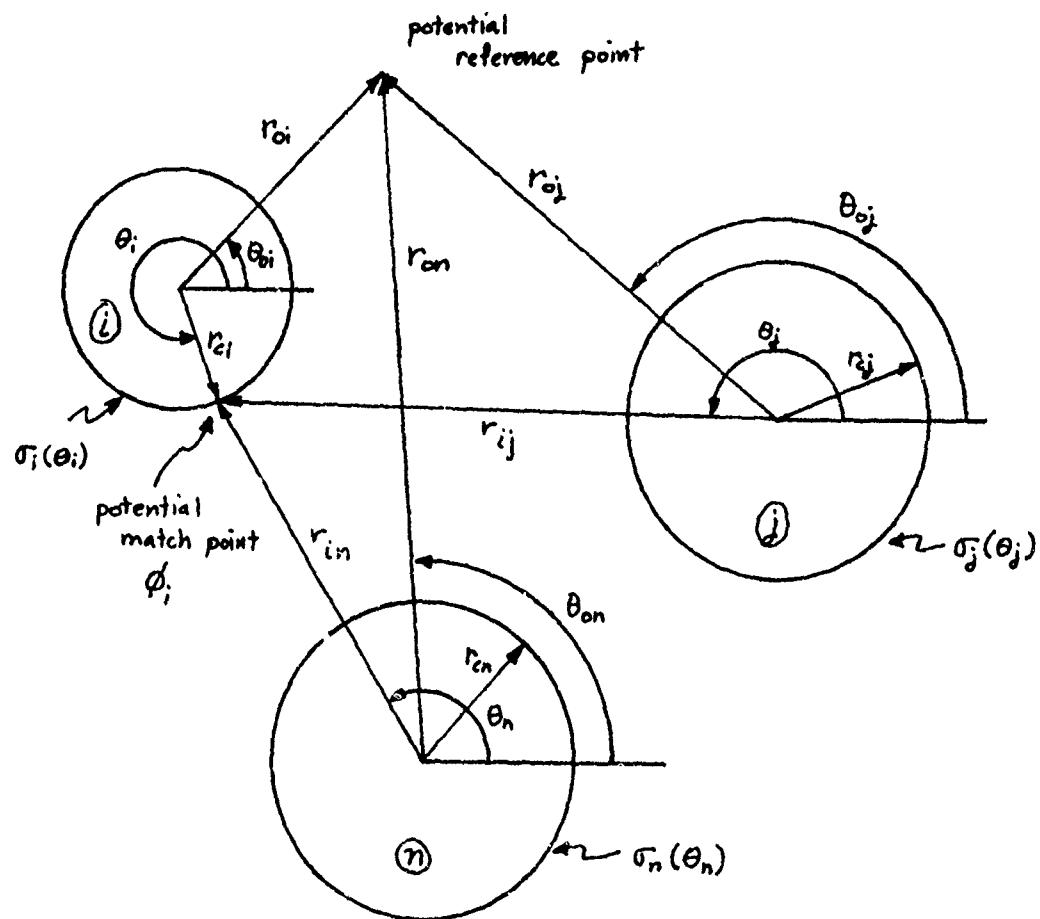
If the cylindrical surface supports a per-unit-length charge distribution of the form

$$\sigma(\theta') = \sigma_0 + \sum_{m=1}^k (\sigma_m \cos m\theta' + \sigma'_m \sin m\theta') \quad (\text{A.3})$$

then the potential $\phi(r, \theta)$ can be obtained as the limiting case of an infinite number of infinitesimal line charges with appropriate weighting given in Eq. (A.3) as [4]



(a)



(b)

Fig. A.1.

$$\begin{aligned}
\phi(r, \theta) = & -\frac{\sigma_0}{2\pi\epsilon_v} \int_0^{2\pi} \ln\left(\frac{d}{d_0}\right) r' d\theta' \\
& - \frac{1}{2\pi\epsilon_v} \sum_{m=1}^k \left\{ \int_0^{2\pi} \sigma_m \cos m\theta' \ln\left(\frac{d}{d_0}\right) r' d\theta' \right. \\
& \left. + \int_0^{2\pi} \sigma'_m \sin m\theta' \ln\left(\frac{d}{d_0}\right) r' d\theta' \right\}. \quad (A.4)
\end{aligned}$$

Substituting the expressions for d and d_0 given in Eq. (A.2) into Eq. (A.4) yield integrals which can be evaluated in closed form. The result is [4]

$$\begin{aligned}
\phi(r, \theta) = & \sigma_0 D_0(r, \theta) - \sigma_0 D_0(r_0, \theta_0) \\
& + \sum_{m=1}^k \left\{ \sigma_m D_m(r, \theta) - \sigma_m D_m(r_0, \theta_0) \right. \\
& \left. + \sigma'_m D'_m(r, \theta) - \sigma'_m D'_m(r_0, \theta_0) \right\} \quad (A.5)
\end{aligned}$$

where

$$D_0(r, \theta) = \begin{cases} -\frac{r' \ln r}{\epsilon_v} & r \geq r' \\ -\frac{r' \ln r'}{\epsilon_v} & r \leq r' \end{cases} \quad (A.6a)$$

$$D_m(r, \theta) = \begin{cases} \frac{(r')^{m+1}}{2\epsilon_v m r^m} \cos m\theta & r \geq r' \\ \frac{(r)^m}{2\epsilon_v m (r')^{m-1}} \cos m\theta & r \leq r' \end{cases} \quad (A.6b)$$

$$D'_m(r, \theta) = \begin{cases} \frac{(r')^{m+1}}{2\epsilon_v m r^m} \sin m\theta & r \geq r' \\ \frac{(r)^m}{2\epsilon_v m (r')^{m-1}} \sin m\theta & r \leq r' \end{cases} \quad (\text{A.6c})$$

The terms $D_0(r_0, \theta_0)$, $D_m(r_0, \theta_0)$ and $D'_m(r_0, \theta_0)$ are the reference potential terms which were omitted in the potential expressions in Chapter II.

Consider a typical system of n circular conductors shown in Fig. A.1(b) bearing per-unit-length charge distributions of the form

$$\sigma_i(\theta_i) = \sigma_{i0} + \sum_{m=1}^{k_i} (\sigma_{im} \cos m\theta_i + \sigma'_{im} \sin m\theta_i) \quad (\text{A.7})$$

A typical expression for the potential at a matchpoint on the i -th conductor in Fig. A.1(b) is

$$\begin{aligned}
\phi_i = & \dots + \sigma_{i0} D_{i0}(r_{ci}, \theta_i) - \sigma_{i0} D_{i0}(r_{0i}, \theta_{0i}) \\
& + \sum_{m=1}^{k_i} \left\{ \sigma_{im} D_{im}(r_{ci}, \theta_i) - \sigma_{im} D_{im}(r_{0i}, \theta_{0i}) \right. \\
& \quad \left. + \sigma'_{im} D'_{im}(r_{ci}, \theta_i) - \sigma'_{im} D'_{im}(r_{0i}, \theta_{0i}) \right\} \\
& \dots + \sigma_{j0} D_{j0}(r_{ij}, \theta_j) - \sigma_{j0} D_{j0}(r_{0j}, \theta_{0j}) \\
& + \sum_{m=1}^{k_j} \left\{ \sigma_{jm} D_{jm}(r_{ij}, \theta_j) - \sigma_{jm} D_{jm}(r_{0j}, \theta_{0j}) \right. \\
& \quad \left. + \sigma'_{jm} D'_{jm}(r_{ij}, \theta_j) - \sigma'_{jm} D'_{jm}(r_{0j}, \theta_{0j}) \right\} \\
& \dots + \sigma_{n0} D_{n0}(r_{in}, \theta_n) - \sigma_{n0} D_{n0}(r_{0n}, \theta_{0n}) \\
& + \sum_{m=1}^{k_n} \left\{ \sigma_{nm} D_{nm}(r_{in}, \theta_n) - \sigma_{nm} D_{nm}(r_{0n}, \theta_{0n}) \right. \\
& \quad \left. + \sigma'_{nm} D'_{nm}(r_{in}, \theta_n) - \sigma'_{nm} D'_{nm}(r_{0n}, \theta_{0n}) \right\}
\end{aligned}
\tag{A.8}$$

If we allow the reference point for the potentials to move to infinity, the reference potential terms or the $\sigma_{im} \cos m\theta_i$ and $\sigma'_{im} \sin m\theta_i$ expansion terms go to zero as is clear from Eq. (A.6b) and (A.6c). If the total per-unit-length charge on the system of conductors is zero, then the reference potential terms due to the constant expansion terms may also be removed. This can be shown in the following manner. The total per-unit-length charge on the i-th

conductor is

$$q_i = \int_0^{2\pi} \sigma_i(\theta_i) r_{ci} d\theta_i$$

$$= 2\pi r_{ci} \sigma_{i0}$$
(A.9)

and the portion of the potential expression in Eq. (A.8) consisting of the reference potential terms due to the constant charge expansion terms is

$$\dots -\sigma_{i0} D_{i0}(r_{0i}, \theta_{0i}) \dots -\sigma_{j0} D_{j0}(r_{0j}, \theta_{0j})$$

$$\dots -\sigma_{n0} D_{n0}(r_{0n}, \theta_{0n})$$
(A.10)

$$= - \sum_{m=1}^n \sigma_{m0} D_{m0}(r_{0m}, \theta_{0m})$$

Utilizing the expression for $D_{m0}(r_{0m}, \theta_{0m})$ of the form given in Eq. (A.6a), Eq. (A.10) can be written as

$$- \sum_{m=1}^n \sigma_{m0} \left(- \frac{r_{cm} \ln(r_{0m})}{\epsilon_v} \right).$$
(A.11)

With the expression for the total per-unit-length charge on the i-th conductor given in Eq. (A.9), Eq. (A.11) can be written as

$$\sum_{m=1}^n q_m \frac{\ln(r_{0m})}{2\pi\epsilon_v}.$$
(A.12)

Requiring the system to be electrically neutral, i.e.,

$$\sum_{m=1}^n q_m = 0 , \quad (A.13)$$

Eq. (A.12) can be written as

$$-\left(\sum_{m=2}^n q_m\right) \frac{\ln(r_{01})}{2\pi\epsilon_v} + \sum_{m=2}^n q_m \frac{\ln(r_{0m})}{2\pi\epsilon_v} . \quad (A.14)$$

By combining associated terms, Eq. (A.14) can be written as

$$\sum_{m=2}^n \frac{q_m}{2\pi\epsilon_v} \ln\left(\frac{r_{0m}}{r_{01}}\right) . \quad (A.15)$$

As the reference potential point moves to infinity, the distance from the centers of the conductors to the reference point become the same, i.e., $r_{01}=r_{02}=\dots=r_{0n}$, and Eq.

(A.15) approaches zero. Therefore, the reference potential terms in the potential expressions may be omitted. Implicit in this is the fact that the potentials, ϕ_i , are with respect to infinity which is permissible only if the net per-unit-length charge on the system is zero, i.e.,

$$\sum_{m=1}^n q_m = 0 . \quad (A.16)$$

APPENDIX B

Listing of Computer Program GETCAP.

The listing will be divided into its individual program parts:

- MAIN - main program controlling input of data
and output of results
- GETCAP - subroutine which computes the per-unit-
length generalized and transmission line
capacitance matrices for ribbon cables
of the specified dimensions
- MINV - subroutine which computes the inverse of
a matrix (from the IBM Scientific
Subroutine Package [16])
- MPC - subroutine which multiplies two matrices
and then multiplies the result by a
constant
- MPRT - subroutine which prints a matrix in
standard matrix format

C	*****	GGG	EEEE	TTTT	CCC	AAA	PPPP	MAIN	10
C	*****	G	E	T	C	A	P	MAIN	20
C	*****	G	E	T	C	A	P	MAIN	30
C	*****	G	EEEE	T	C	AAAA	PPPP	MAIN	40
C	*****	G	E	T	C	A	P	MAIN	50
C	*****	G	E	T	C	A	P	MAIN	60
C	*****	G	E	T	C	A	P	MAIN	70
C	*****	GGG	EEEE	T	CCC	A	P	MAIN	80
C	*****							MAIN	90
C	*****							MAIN	100
C	*****							MAIN	110
C	*****							MAIN	120
C	*****							MAIN	130
C	*****							MAIN	140
C	*****							MAIN	150
C	*****							MAIN	160
C	*****							MAIN	170
C	*****							MAIN	180
C	*****							MAIN	190
C	*****							MAIN	200
C	*****							MAIN	210
C	*****							MAIN	220
C	*****							MAIN	230
C	*****							MAIN	240
C	*****							MAIN	250
C	*****							MAIN	260
C	*****							MAIN	270
C	*****							MAIN	280
C	*****							MAIN	290
C	*****							MAIN	300
C	*****							MAIN	310
C	*****							MAIN	320
C	*****							MAIN	330
C	*****							MAIN	340
C	*****							MAIN	350
C	*****							MAIN	360

PROGRAM TO COMPUTE THE GENERALIZED AND TRANSMISSION
 LINE CAPACITANCE MATRICES FOR RIBBON OR
 "FLAT-PACK" CABLES

WRITTEN BY ARTHUR E. FEATHER
 ELECTRICAL ENGINEERING DEPARTMENT
 UNIVERSITY OF KENTUCKY
 LEXINGTON, KENTUCKY 40506

```

C.....MAIN 370
C.....MAIN 380
C.....MAIN 390
C.....MAIN 400
C.....MAIN 410
C.....MAIN 420
C.....MAIN 430
C.....MAIN 440
C.....MAIN 450
C.....MAIN 460
C.....MAIN 470
C.....MAIN 480
C.....MAIN 490
C.....MAIN 500
C.....MAIN 510
C.....MAIN 520
C.....MAIN 530
C.....MAIN 540
C.....MAIN 550
C.....MAIN 560
C.....MAIN 570
C.....MAIN 580
C.....MAIN 590
C.....MAIN 600
C.....MAIN 610
C.....MAIN 620
C.....MAIN 630
C.....MAIN 640
C.....MAIN 650
C.....MAIN 660
C.....MAIN 670
C.....MAIN 680
C.....MAIN 690
C.....MAIN 700
C.....MAIN 710
C.....MAIN 720

      DIMENSIONING INFORMATION:
      C(NW**2)
      C1((NW-1)**2)
      D(NF**2)
      D1((NW*NF)**2)
      LT(2*NF*NW)
      SCR((NF*NW+1)/2)
      WHERE:
      NW IS THE NUMBER OF WIRES
      NF IS THE TOTAL NUMBER OF FOURIER COEFF.
      I.E., NF=NFC+NFD

      THIS DIMENSION PROVIDES FOR UP TO 20 WIRES AND UP TO
      10 FOURIER COEFFICIENTS PER WIRE
      DIMENSION C(400),C1(361),D(10,10),D1(200,200),LT(400),SCR(100)

      DIMENSION C(25),C1(16),D(20,20),D1(100,100),LT(200),SCR(50)

      SCR AND LT ARE SCRATCH VECTORS OF DIFFERENT TYPE BUT CAN
      SHARE STORAGE LOCATIONS
      EQUIVALENCE (SCR(1),LT(1))
      WRITE(6,46)
      WRITE(6,47)
      READ(5,30,END=11,ERR=9)NW,NFC,NFD
      WRITE(6,31)NW,NFC,NFD

      CALL FOR ERROR IF NW IS LESS THAN 2

      IF (NW-1)2,2,3
      WRITE(6,32)
      GO TO 10

```

C				MAIN 730
C				MAIN 740
C				MAIN 750
3				MAIN 760
4				MAIN 770
				MAIN 780
5				MAIN 790
				MAIN 800
				MAIN 810
				MAIN 820
C				MAIN 830
C				MAIN 840
C				MAIN 850
				MAIN 860
6				MAIN 870
				MAIN 880
				MAIN 890
				MAIN 900
7				MAIN 910
8				MAIN 920
				MAIN 930
9				MAIN 940
10				MAIN 950
				MAIN 960
				MAIN 970
11				MAIN 980
				MAIN 990
12				MAIN1000
				MAIN1010
13				MAIN1020
14				MAIN1030
15				MAIN1040
C				MAIN1050
C				MAIN1060
C				MAIN1070
16				MAIN1080

		CALL FOR ERROR IF NFC IS GREATER THAN NFD
		IF (NFD-NFC) 4, 5, 5
		WRITE(6, 33) NFC
		NFD=NFC
		READ(5, 34, END=8, ERR=7) RC, RD, SEP, ER
		WRITE(6, 35) RC, RD, SEP, ER
		READ(5, 30, END=6, ERR=6) IREF, IOPT
		WRITE(6, 36) IREF, IOPT
		MAKE SURE IREF IS WITHIN RANGE
		IF (IREF-1) 12, 16, 13
		WRITE(6, 39)
		IREF=1
		IOPT=0
		GO TO 21
		READ(6, 41, END=8)
		WRITE(6, 40)
		GO TO 1
		WRITE(6, 43)
		READ(5, 41, END=11)
		READ(5, 41, END=11)
		GO TO 1
		WRITE(6, 42)
		STOP
		IREF=1
		GO TO 15
		IF (NW-IREF) 14, 16, 16
		IREF=NW
		WRITE(6, 37) IREF
		CHECK TO SEE IF IOPT IS WITHIN RANGE
		IPCH=0

```

17      IOPT10=IOPT-10
      IF (IOPT10) 18,17,17
      IPCH=1
      IOPT=IOPT10
18      IF (IOPT-1) 19,24,21
19      IF (IOPT) 20,24,24
20      IOPT=0
21      IF (IOPT-2) 23,24,22
22      IOPT=2
23      WRITE(6,38) IOPT
      C
      C      GENERATE CONSTANTS NEEDED TO PASS TO SUBROUTINE
      C
      NF=NFC
24      IF (IOPT-2) 25,26,25
      NF=NF+NFD
25      CALL GETCAP (NW,NFC,NFD,NF,RC,RD,SEP,ER,IREF,IOPT,C,C1,D,D1,SCR,LT)
26      C
      C      CHECK FOR ERROR RETURN
      C
      IF (NW) 27,1,27
      C
      C      RESULTS HAVE BEEN CALCULATED- -MATRICES ARE PRINTED OUT
      C
      CALL MPRT (C,NW,NW,'GENERALIZED CAPACITANCE',23)
      NW1=NW-1
27      CALL MPRT (C1,NW1,NW1,'TRANSMISSION LINE CAPACITANCE',29)
      IF (IPCH) 1,1,28
28      DO 29 I=1,NW1
      II=(I-1)*NW1
      DO 29 J=1,I
29      WRITE(7,44) J,I,C1(II+J)
      WRITE(6,45)
      GO TO 1
      C
      C      ERROR MESSAGES AND OUTPUT INFORMATION

```

```

C 30      FORMAT(3I3)
31      FORMAT('1',100('*'),//,6X,'GENERALIZED AND TRANSMISSION LINE CAPACITANCE MATRICES OF "FLAT-PACK" CABLE',//,6X,'CONDITIONS:',//,11X,I3MAIN1480
32      1TANCE WIRES, 'I3,' FOURIER COEFFICIENTS AROUND THE CONDUCTOR, AND ',MAIN1490
33      1I3,' AROUND THE DIELECTRIC',//)
34      FORMAT(/, ' ***ERROR*** NUMBER OF WIRES LESS THAN TWO. WILL SKIP THIS PROBLEM.',//)
35      FORMAT(/, ' ***ERROR*** NUMBER OF COEFFICIENTS AROUND CONDUCTOR GIMAIN1530
36      1VEN GREATER THAN NUMBER AROUND DIELECTRIC.',//,14X,'BOTH SET EQUAL MAIN1540
37      1TO 'I3',',',//)
38      FORMAT(4E10.0)
39      FORMAT(/,6X,'PHYSICAL CHARACTERISTICS:',//,10X,'CONDUCTOR RADIUS= 'MAIN1570
40      1,1PE11.4,' (METERS)',//,10X,'DIELECTRIC RADIUS= ',E11.4,' (METERS)'MAIN1580
41      1,/,10X,'CENTER-TO-CENTER SEPARATION= ',E11.4,' (METERS)',//,10X,'REMAIN1590
42      1LATIVE DIELECTRIC CONSTANT= ',E11.4,/)
43      FORMAT(/,6X,'REFERENCE CONDUCTOR IS NO. ',I3,/,6X,'OPTION NUMBER IMAIN1610
44      1S ',I3,/,/,',100('*'),)
45      FORMAT(/, ' ***ERROR*** REFERENCE CONDUCTOR GIVEN OUT OF RANGE. SMAIN1630
46      1ET EQUAL TO ',I3,.',//)
47      FORMAT(/, ' ***ERROR*** OPTION NUMBER OUT OF RANGE. SET EQUAL TO MAIN1650
48      1',I3,/)
49      FORMAT(/, ' ***ERROR*** CARD WITH REFERENCE AND OPTION IN ERROR ORMAIN1670
50      1 MISSING. DEFAULT IREF=1, IOPT=0.',//)
51      FORMAT(/, ' ***ERROR*** PHYSICAL CHARACTERISTICS CARD MISSING OR IMAIN1690
52      1N ERROR. WILL SKIP THIS PROBLEM.',//)
53      FORMAT(' ',)
54      FORMAT(/, ' ***ERROR*** CARDS MISSING. END OF RUN.',//)
55      FORMAT(/, ' ***ERROR*** FIRST CARD IN ERROR. WILL SKIP THIS PROBLEMMAIN1730
56      1EM.',//)
57      FORMAT('C',2(2X,I3),1PE15.6)
58      FORMAT(/,/, ' UPPER TRIANGLE OF TRANSMISSION LINE CAPACITANCE MATRIMAIN1750
59      1X WAS PUNCHED',//)
60      FORMAT('1',/,/,2(10X,70('*'),//,22X,'GGG EEEEE TTTT CCC MAIN1770
61      1AAA PPPP',//,21X,'G G E',8X,'T C C A A A P P',/,21X,'MAIN1790
62      1G E',8X,'T C A A A P P',/,21X,'G GG EEEE T MAIN1800

```

```

1 C      AAAA PPPP',/,21X,'G G E',8X,'T C A A P',/MAIN1810
1,21X,'G G E',8X,'T C A A P',/,22X,'GGG EEEE T MAIN1820
1 CCC A A P',/)))
47 FORMAT(18X,' GENERALIZED AND TRANSMISSION LINE CAPACITANCE MATRICEMAIN1830
1S',/,'+',18X,' ',14X,' ',17X,' ',/,35X,'OF RIBBON CABLES',/,/,MAIN1850
1/,11X,70(' '),/,/,30X,'B A T C H R U N')
END
MAIN1860
MAIN1870

```



```

C      DELTC=PI/(2.0E0*NFC)
      DEJTD=PI/(2.0E0*NFD)
      CHECK FOR NO DIELECTRIC CASE
      IOPT2=IOPT-2
      IF (IOPT2) 2,1,2
1     NF=NFC
      IOPT=1
C      ORDER OF "D1" MATRIX
2     LD=NF*NW
C      ORDER OF THE PARTITIONS OF THE "D1" MATRIX
      LP1=LD/2
      LP2=LD-LP1
      LS1=LP1*LP1
      LX=LP1*LP2
C
C*****  SCALE INPUT DATA WITH RC BETWEEN 0.1 AND 1.0 TO PREVENT
C*****  COMPUTER NUMERICAL INCONSISTENCIES
C
      J=-ALOG10(RC)
      A1=10**J
      RC=RC*A1
      RD=RD*A1
      CCSEP=CCSEP*A1
C
C*****  COMPUTATION OF THE OFF-DIAGONAL "D" SUBMATRICES *****
C
      IW=1
      MW=1
      LU=NW
      GO TO 4
C
C      MW IS THE PRESENT MATCH WIRE
C      IW INDICATES FROM WHICH END OF THE CABLE THE PRESENT POSITION
C      IS COMPUTED.
C      MPP IS THE PRESENT MATCH POINT ON THE BOUNDARY
C      XSEP IS THE SEPARATION DISTANCE FROM MW TO IW

```

```

GETC 730
GETC 740
GETC 750
GETC 760
GETC 770
GETC 780
GETC 790
GETC 800
GETC 810
GETC 820
GETC 830
GETC 840
GETC 850
GETC 860
GETC 870
GETC 880
GETC 890
GETC 900
GETC 910
GETC 920
GETC 930
GETC 940
GETC 950
GETC 960
GETC 970
GETC 980
GETC 990
GETC1000
GETC1010
GETC1020
GETC1030
GETC1040
GETC1050
GETC1060
GETC1070
GETC1080

```

```

C 3      IW=NW
      MW=0
      LU=NW-1
      MW=MW+1
      XSEP=(IW-MW)*CCSEP
C
C      THIS SECTION COMPUTES THE POTENTIALS ON THE MATCH WIRE DUE TO
C      THE WIRE AND DIELECTRIC OF THE END WIRE
C
      DO 13 MPP=1,NFC
      ANG=(MPP-1)*AC+DELTG
      CANG= COS(ANG)
      SANG= SIN(ANG)
      MP= CMLX(RC*CANG+XSEP,RC*SANG)
      RO= CABS(MP)
      AO= ATAN2(AIMAG(MP), REAL(MP))
      A1=ALOG(RO)
      D(MPP,1)=-RC*A1
      IF(IOPT2)5,6,5
5      D(MPP,NFC1)=-RD*A1
6      A2=RC
      A3=RD
      A11=RO*2.0E0
      IF(NCC)10,10,7
7      DO 9 J=1,NCC
      J1=J+1
      J2=NFC+J1
      A1= COS(J*AO)/(J*A11)
      A2=A2*RC
      A3=A3*RD
      A11=A11*RO
      IF(IOPT2)8,9,8
8      D(MPP,J2)=A1*A3
9      D(MPP,J1)=A1*A2
      IF(IOPT2)10,13,10
      GETC1090
      GETC1100
      GETC1110
      GETC1120
      GETC1130
      GETC1140
      GETC1150
      GETC1160
      GETC1170
      GETC1180
      GETC1190
      GETC1200
      GETC1210
      GETC1220
      GETC1230
      GETC1240
      GETC1250
      GETC1260
      GETC1270
      GETC1280
      GETC1290
      GETC1300
      GETC1310
      GETC1320
      GETC1330
      GETC1340
      GETC1350
      GETC1360
      GETC1370
      GETC1380
      GETC1390
      GETC1400
      GETC1410
      GETC1420
      GETC1430
      GETC1440

```

```

10 IF (NFC-NCD) 11,11,13
11 DO 12 J=NFC,NCD
    J2=J+NFC1
    A1= COS (J*AO) / (J*A11)
    A3=A3*RD
    A11=A11*RO
12 D(MPP,J2)=A1*A3
13 CONTINUE
    IF (IOPT2) 14,22,14
C
C      THIS SECTION COMPUTES THE ELECTRIC FIELD COMPONENTS ON THE
C      MATCH WIRE DUE TO BOTH THE CONDUCTOR AND DIELECTRIC OF THE
C      END WIRE
C
14 DO 21 MPP=1,NFD
    ANG=(MPP-1)*AD+DELTD
    CANG= COS (ANG)
    SANG= SIN (ANG)
    MP= CMLX (RD*CANG+XSEP, RD*SANG)
    RO= CABS (MP)
    AO= ATAN2 (AIMAG (MP), REAL (MP))
    A1=(CANG* COS (AO)+SANG* SIN (AO))*ER1/RO
    JJ=NFC+MPP
    D (JJ,1)=RC*A1
15 D (JJ,NFC1)=RD*A1
    A2=RC
    A3=RD
    A11=RO*2.0E0
    IF (NCC) 18,18,16
16 DO 17 J=1,NCC
    J1=J+1
    J2=NFC+J1
    A2=A2*RC
    A3=A3*RD
    A11=A11*RO
    A1=(CANG* COS (J1*AO)+SANG* SIN (J1*AO))*ER1/A11
    GETC1450
    GETC1460
    GETC1470
    GETC1480
    GETC1490
    GETC1500
    GETC1510
    GETC1520
    GETC1530
    GETC1540
    GETC1550
    GETC1560
    GETC1570
    GETC1580
    GETC1590
    GETC1600
    GETC1610
    GETC1620
    GETC1630
    GETC1640
    GETC1650
    GETC1660
    GETC1670
    GETC1680
    GETC1690
    GETC1700
    GETC1710
    GETC1720
    GETC1730
    GETC1740
    GETC1750
    GETC1760
    GETC1770
    GETC1780
    GETC1790
    GETC1800

```

```

17 D(JJ,J1)=A1*A2
18 D(JJ,J2)=A1*A3
19 IF(NFC-NCD) 19,19,21
DO 20 J=NFC,NCD
J1=J+1
J2=NFC1+J
A3=A3*RD
A11=A11*RO
A1=(CANG* COS(J1*AO)+SANG* SIN(J1*AO))*ER1/A11
20 D(JJ,J2)=A1*A3
21 CONTINUE
C
C THIS SECTION TELLS WHERE TO PUT THE SUBMATRIX "D" IN THE LARGE
C MATRIX "D1"
C
MM=IW
NN=MW
GO TO 89
23 IF(IW-1) 24,24,26
24 IF(NW-NN) 28,28,25
25 MM=MM+1
NN=NN+1
GO TO 89
26 IF(NN-1) 28,28,27
27 MM=MM-1
NN=NN-1
GO TO 89
28 IF(MW-LU) 4,29,29
29 IF(IW-1) 3,3,30
C
C***** COMPUTATION OF DIAGONAL "D" MATRIX *****
C
C THIS SECTION COMPUTES THE POTENTIALS ON THE CONDUCTORS DUE TO
THEMSELVES AND ALSO THE DIELECTRIC SURROUNDING THEM
C
GETC1810
GETC1820
GETC1830
GETC1840
GETC1850
GETC1860
GETC1870
GETC1880
GETC1890
GETC1900
GETC1910
GETC1920
GETC1930
GETC1940
GETC1950
GETC1960
GETC1970
GETC1980
GETC1990
GETC2000
GETC2010
GETC2020
GETC2030
GETC2040
GETC2050
GETC2060
GETC2070
GETC2080
GETC2090
GETC2100
GETC2110
GETC2120
GETC2130
GETC2140
GETC2150
GETC2160

```



```

45      A3=1.0E0
      IF(NCC)47,47,45
      DO 46 J=1,NCC
      J1=J+1
      A2= COS(J*ANG)/2.0E0
      D(MPP,J1)=ER1*A3*A12*A2
      D(MPP,J1+NFC)=-ER2*A2
      A3=A3*A1
      IF(NFC-NCD)48,48,50
      DO 49 J=NFC,NCD
      D(MPP,J+1+NFC)=-ER2* COS(J*ANG)/2.0E0
      CONTINUE
      C
      C      THIS SECTION INSERTS THE SUBMATRIX ON THE DIAGONAL OF THE
      C      LARGE "D1" MATRIX NW TIMES
      C
      MM=1
      NN=1
      GO TO 89
      MM=MM+1
      NN=NN+1
      IF (NW-MM) 53,89,89
      C
      C***** INVERSION OF "D1" AND COMPUTATION OF THE GENERALIZED *****
      C      CAPACITANCE MATRIX CG
      C
      C      IF IOPT=1, A FULL MATRIX INVERSION OF THE "D1" MATRIX IS DONE
      C      THEN THE TERMS OF THE GENERALIZED CAPACITANCE MATRIX ARE
      C      COMPUTED
      C
      C      A3=2.0E0*PI*EPS
      53      IF(IOPT-1)61,54,61
      54      CALL MINV(D1,LD,DET,LT(1),LT(LD+1))
      ISTD=1
      IF(DET)55,102,55
      55      DO 60 I=1,NW

```

```

GETC2530
GETC2540
GETC2550
GETC2560
GETC2570
GETC2580
GETC2590
GETC2600
GETC2610
GETC2620
GETC2630
GETC2640
GETC2650
GETC2660
GETC2670
GETC2680
GETC2690
GETC2700
GETC2710
GETC2720
GETC2730
GETC2740
GETC2750
GETC2760
GETC2770
GETC2780
GETC2790
GETC2800
GETC2810
GETC2820
GETC2830
GETC2840
GETC2850
GETC2860
GETC2870
GETC2880

```

```

C      IROW=(I-1)*NF+1
C      DO 60 J=1,NW
C      IL=(J-1)*LD*NF+IROW
C      A1=0.0E0
C      A2=0.0E0
C      DO 57 K=1,NFC
C      TERMS OF THE CAPACITANCE MATRIX ARE DERIVED FROM THE SUM OF
C      CERTAIN ROWS OF THE MATRIX
C
C      A1=A1+D1(IL)
C      IF(IOPT2)56,57,56
C      A2=A2+D1(IL+NFC)
C      IL=IL+LD
C      IF(IOPT2)59,58,59
C      CG(I,J)=A1*RC*A3
C      GO TO 60
C      CG(I,J)=(A1*RC+A2*RD)*A3
C      CONTINUE
C      GO TO 79
C
C      WHEN IOPT DOES NOT EQUAL 1, THE PROGRAM PARTITIONS THE "D1"
C      MATRIX AND COMPUTES ONLY HALF OF THE INVERSE OF "D1". SINCE
C      THE GENERALIZED CAPACITANCE MATRIX IS IDEALLY "SUPER-SYMMETRIC"
C      ONLY A FEW OF THESE TERMS ARE ACTUALLY COMPUTED. THE REST ARE
C      COPIED FROM THOSE WHICH ARE DONE
C
C      NP=1
C      NQ=NP+LS1
C      NR=NQ+LX
C      NS=NR+LX
C      CALL MINV(D1(NS),LP2,DET,LT(1),LT(LP2+1))
C      ISTEP=2
C      IF(DET)62,102,62
C      CALL MPC(D1(NS),D1(NR),D1(NS),SCR,LP2,LP2,LP1,1.0E0)
C      CALL MPC(D1(NQ),D1(NS),D1(NQ),SCR,LP1,LP2,LP1,1.0E0)
C
C      61
C      62

```

63	II=0		GETC3250
64	DO 64 J=1,LP1		GETC3260
	DO 63 I=1,LP1		GETC3270
	I2=II+I		GETC3280
	D1(I2)=D1(I2)-D1(I2+LS1)		GETC3290
	II=II+LP1		GETC3300
	CALL MINV(D1(NP),LP1,DET,LT(1),LT(LP1+1))		GETC3310
	ISTP=3		GETC3320
	IF(DET) 65,102,65		GETC3330
65	CALL MPC(D1(NS),D1(NP),D1(NR),SCR,LP2,LP1,LP1,-1.0E0)		GETC3340
	IA=(LD-1)*LP1		GETC3350
	IL=1		GETC3360
	IH=NW		GETC3370
66	ICOL=(IL-1)*NF		GETC3380
	DO 73 I=IL,IH		GETC3390
	IROW=(I-1)*NF+1		GETC3400
	IROW1=IROW+NFC		GETC3410
	IF(LP1-IROW) 69,57,67		GETC3420
67	INC=LP1		GETC3430
	IS=ICOL*LP1+IROW		GETC3440
	IF(LP1-IROW1) 70,68,68		GETC3450
68	INC1=LP1		GETC3460
	IS1=IS+NFC		GETC3470
	GO TO 71		GETC3480
69	INC=LP2		GETC3490
	IS=IA+ICOL*LP2+IROW		GETC3500
70	INC1=LP2		GETC3510
	IS1=IA+ICOL*LP2+IROW1		GETC3520
71	A1=0.0E0		GETC3530
	A2=0.0E0		GETC3540
	DO 72 J=1,NFC		GETC3550
	A1=A1+D1(IS)		GETC3560
	A2=A2+D1(IS1)		GETC3570
	IS=IS+INC		GETC3580
72	IS1=IS1+INC1		GETC3590
73	CG(I,IL)=(A1*RC+A2*RD)*A3		GETC3600

```

C      IL=IL+1
C      IH=IH-1
C      IF (IH-IL) 74,66,66
C
C      THIS SECTION FILLS OUT THE REST OF THE "C" MATRIX
C
C      NW2=NW/2
C      NP=NW+1
C      NS=2
C      NQ=NW
C      DO 76 I=1,NW2
C      DO 75 J=NS,NQ
C      CG(NQ,J)=CG(NP-J,I)
C      NS=NS+1
C      NQ=NQ-1
C      NS=2
C      DO 78 I=1,NW1
C      DO 77 J=NS,NW
C      CG(I,J)=CG(J,I)
C      NS=NS+1
C
C      ***** THIS SECTION COMPUTES THE TRANSMISSION LINE CAPACITANCE
C      MATRIX FROM THE GENERALIZED CAPACITANCE MATRIX
C
C      A1 IS THE MATRIX SUM
C      A2 IS THE ROWSUM
C      A3 IS THE COLUMN SUM-- STORED IN D1(1-NW1)
C
C      A1=0.0E0
C      DO 80 I=1,NW
C      A1=A1+CG(IREF,I)
C      II=1
C      DO 84 I=1,NW
C      IF (I-IREF) 81,84,81
C      A2=0.0E0
C      A3=0.0E0
C
C      79
C      80
C      81

```

```

GETC3610
GETC3620
GETC3630
GETC3640
GETC3650
GETC3660
GETC3670
GETC3680
GETC3690
GETC3700
GETC3710
GETC3720
GETC3730
GETC3740
GETC3750
GETC3760
GETC3770
GETC3780
GETC3790
GETC3800
GETC3810
GETC3820
GETC3830
GETC3840
GETC3850
GETC3860
GETC3870
GETC3880
GETC3890
GETC3900
GETC3910
GETC3920
GETC3930
GETC3940
GETC3950
GETC3960

```

```

DO 82 J=1,NW
A2=A2+CG(I,J)
A3=A3+CG(J,I)
A1=A1+A2
D1(II)=A3
DO 83 J=II,NW12,NW1
CTL(J)=A2
II=II+1
CONTINUE
II=1
JJ=1
DO 88 J=1,NW
IF(J-IREF)85,88,85
DO 87 I=1,NW
IF(I-IREF)86,87,86
CFL(II)=CG(I,J)-CTL(II)*D1(JJ)/A1
II=II+1
CONTINUE
JJ=JJ+1
CONTINUE
C
C
C
C
C
C
      AFTER THE PER-UNIT-LENGTH GENERALIZED AND TRANSMISSION LINE
      CAPACITANCE MATRICES HAVE BEEN CALCULATED, CONTROL RETURNS
      TO THE CALLING PROGRAM
      RETURN
C
C***** THIS SECTION INSERTS THE SMALL "D" MATRICES INTO THE *****
          LARGE "D1" MATRIX
C
C
C
C
C
C
      WHEN IOPT IS 1, THE MATRICES ARE INSERTED DIRECTLY.
      WHEN IOPT IS 0, "D1" IS PARTITIONED AND "D" IS INSERTED
      ACCORDINGLY.
C
C
C
      IF(IOPT-1)90,99,90
      ICB=(NN-1)*NF

```

```

91  IRB=(MM-1)*NF
    IRX=IRB-LP1
    IF (IRX) 91,94,94
    NP=ICB*LP1+IRB
    NQ=MIN0(NF,-IRX)
    DO 93 J=1,NF
    DO 92 I=1,NQ
    D1(NP+I)=D(I,J)
    NP=NP+LP1
    NP=-NQ
    IF (NQ-NF) 95,98,98
    NQ=0
    NP=IRX
    NQ=NQ+1
    NP=NP+LD*LP1+ICB*LP2
    DO 97 J=1,NF
    DO 96 I=NQ,NF
    D1(NP+I)=D(I,J)
    NP=NP+LP2
    IF (MM-NN) 23,52,23
    NP=NF*(MM-1+(NN-1)*LD)
    DO 101 I=1,NF
    DO 100 J=1,NF
    D1(NP+J)=D(J,I)
    NP=NP+LD
    GO TO 98
C ***** ERROR RETURN *****
C
102 WRITE(6,103)ISTP
    NW=0
    RETURN
103 FORMAT(' ** SINGULAR MATRIX AT STEP ',I1)
    END
GETC4330
GETC4340
GETC4350
GETC4360
GETC4370
GETC4380
GETC4390
GETC4400
GETC4410
GETC4420
GETC4430
GETC4440
GETC4450
GETC4460
GETC4470
GETC4480
GETC4490
GETC4500
GETC4510
GETC4520
GETC4530
GETC4540
GETC4550
GETC4560
GETC4570
GETC4580
GETC4590
GETC4600
GETC4610
GETC4620
GETC4630
GETC4640
GETC4650
GETC4660

```

C	MINV	10
C	MINV	20
C	MINV	30
C	MINV	40
C	MINV	50
C	MINV	60
C	MINV	70
C	MINV	80
C	MINV	90
C	MINV	100
C	MINV	110
C	MINV	120
C	MINV	130
C	MINV	140
C	MINV	150
C	MINV	160
C	MINV	170
C	MINV	180
C	MINV	190
C	MINV	200
C	MINV	210
C	MINV	220
C	MINV	230
C	MINV	240
C	MINV	250
C	MINV	260
C	MINV	270
C	MINV	280
C	MINV	290
C	MINV	300
C	MINV	310
C	MINV	320
C	MINV	330
C	MINV	340
C	MINV	350
C	MINV	360

.....

SUBROUTINE MINV

PURPOSE
INVERT A MATRIX

USAGE
CALL MINV(A,N,D,L,M)

DESCRIPTION OF PARAMETERS
A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY
RESULTANT INVERSE.
N - ORDER OF MATRIX A
D - RESULTANT DETERMINANT
L - WORK VECTOR OF LENGTH N
M - WORK VECTOR OF LENGTH N

REMARKS
MATRIX A MUST BE A GENERAL MATRIX

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
NONE

METHOD
THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT
IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT
THE MATRIX IS SINGULAR.

.....

SUBROUTINE MINV (A,N,D,L,M)
DIMENSION A(1),L(1),M(1)

SEARCH FOR LARGEST ELEMENT

MINV 370
MINV 380
MINV 390
MINV 400
MINV 410
MINV 420
MINV 430
MINV 440
MINV 450
MINV 460
MINV 470
MINV 480
MINV 490
MINV 500
MINV 510
MINV 520
MINV 530
MINV 540
MINV 550
MINV 560
MINV 570
MINV 580
MINV 590
MINV 600
MINV 610
MINV 620
MINV 630
MINV 640
MINV 650
MINV 660
MINV 670
MINV 680
MINV 690
MINV 700
MINV 710
MINV 720

C
D=1.0
NK=-N
DO 19 K=1,N
NK=NK+N
L(K)=K
M(K)=K
FK=NK+K
BIGA=A(KK)
DO 3 J=K,N
IZ=N*(J-1)
DO 3 I=K,N
IJ=IZ+I
IF(ABS(BIGA)-ABS(A(IJ))) 2,3,3
BIGA=A(IJ)
L(K)=I
M(K)=J
CONTINUE
INTERCHANGE ROWS
J=L(K)
IF(J-K) 6,6,4
KI=K-N
DO 5 I=1,N
KI=KI+N
HOLD=-A(KI)
JI=KI-K+J
A(KI)=A(JI)
A(JI)=HOLD
INTERCHANGE COLUMNS
I=M(K)
IF(I-K) 9,9,7
JP=N*(I-1)

1
2
3
C
C
C
4
5
C
C
C
6
7

MINV 730
MINV 740
MINV 750
MINV 760
MINV 770
MINV 780
MINV 790
MINV 800
MINV 810
MINV 820
MINV 830
MINV 840
MINV 850
MINV 860
MINV 870
MINV 880
MINV 890
MINV 900
MINV 910
MINV 920
MINV 930
MINV 940
MINV 950
MINV 960
MINV 970
MINV 980
MINV 990
MINV1000
MINV1010
MINV1020
MINV1030
MINV1040
MINV1050
MINV1060
MINV1070
MINV1080

```

DO 8 J=1,N
JK=NK+J
JI=JP+J
HOLD=-A(JK)
A(JK)=A(JI)
A(JI)=HOLD

      DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
      CONTAINED IN BIGA)

IF(BIGA) 11,10,11
D=0.0
RETURN
DO 13 I=1,N
IF(I-K) 12,13,12
IK=NK+I
A(IK)=A(IK)/(-BIGA)
CONTINUE

      REDUCE MATRIX

DO 16 I=1,N
IK=NK+I
HOLD=A(IK)
IJ=I-N
DO 16 J=1,N
IJ=IJ+N
IF(I-K) 14,16,14
IF(J-K) 15,16,15
KJ=IJ-I+K
A(IJ)=HOLD*A(KJ)+A(IJ)
CONTINUE

      DIVIDE ROW BY PIVOT

KJ=K-N

```

8
C
C
C
C
9
10
11
12
13
C
C
C

14
15
16
C
C
C

17	DO 18 J=1,N	MINV1090
18	KJ=KJ+N	MINV1100
C	IF (J-K) 17,18,17	MINV1110
C	A(KJ)=A(KJ)/BIGA	MINV1120
C	CONTINUE	MINV1130
C	PRODUCT OF PIVOTS	MINV1140
C	D=D*BIG A	MINV1150
C	REPLACE PIVOT BY RECIPROCAL	MINV1160
C	A(KK)=1.0/BIG A	MINV1170
19	CONTINUE	MINV1180
C	FINAL ROW AND COLUMN INTERCHANGE	MINV1190
C	K=N	MINV1200
20	K=(K-1)	MINV1210
21	IF (K) 27,27,21	MINV1220
22	I=L(K)	MINV1230
	IF (I-K) 24,24,22	MINV1240
	JQ=N*(K-1)	MINV1250
	JR=N*(I-1)	MINV1260
	DO 23 J=1,N	MINV1270
	JK=JQ+J	MINV1280
	HOLD=A(JK)	MINV1290
	JI=JR+J	MINV1300
23	A(JK)=-A'JI)	MINV1310
24	A(JI) =HOLD	MINV1320
	J=M(K)	MINV1330
25	IF (J-K) 20,20,25	MINV1340
	KI=K-N	MINV1350
	DO 26 I=1,N	MINV1360
	KI=KI+N	MINV1370
	HOLD=A(KI)	MINV1380
		MINV1390
		MINV1400
		MINV1410
		MINV1420
		MINV1430
		MINV1440

MINV1450
MINV1460
MINV1470
MINV1480
MINV1490
MINV1500

JI=KI-K+J
A(KI)=-A(JI)
A(JI)=HOLD
GO TO 20
RETURN
END

26

27

```

C ***** MPC 10
C ***** MPC 20
C ***** MPC 30
C ***** MPC 40
C ***** MPC 50
C ***** MPC 60
C ***** MPC 70
C ***** MPC 80
C ***** MPC 90
C ***** MPC 100
C ***** MPC 110
C ***** MPC 120
C ***** MPC 130
C ***** MPC 140
C ***** MPC 150
C ***** MPC 160
C ***** MPC 170
C ***** MPC 180
C ***** MPC 190
C ***** MPC 200
C ***** MPC 210
C ***** MPC 220
C ***** MPC 230
C ***** MPC 240
C ***** MPC 250
C ***** MPC 260
C ***** MPC 270
C ***** MPC 280
C ***** MPC 290
C ***** MPC 300
C ***** MPC 310
C ***** MPC 320
C ***** MPC 330
C ***** MPC 340
C ***** MPC 350
C ***** MPC 360

SUBROUTINE MPC

PURPOSE
  TO MULTIPLY TWO GENERAL MATRICES, AND THEN MULTIPLY THAT
  RESULT BY A CONSTANT

USAGE
  CALL MPC(A,B,R,S,L,M,N,C)

DESCRIPTION OF PARAMETERS
  A - FIRST MATRIX, L BY M
  B - SECOND MATRIX, M BY N
  R - RESULTANT MATRIX (CAN BE SAME AS A), L BY N
  S - WORKING VECTOR, DIMENSION M
  L - NUMBER OF ROWS IN A AND R
  M - NUMBER OF COLUMNS IN A AND ROWS IN B
  N - NUMBER OF COLUMNS IN B AND R
  C - CONSTANT BY WHICH R IS MULTIPLIED

REMARKS
  NOTE THAT A AND R CAN BE THE SAME MATRIX IF DIMENSIONED
  CORRECTLY

C ***** MPC 270
C ***** MPC 280
C ***** MPC 290
C ***** MPC 300
C ***** MPC 310
C ***** MPC 320
C ***** MPC 330
C ***** MPC 340
C ***** MPC 350
C ***** MPC 360

SUBROUTINE MPC(A,B,R,S,L,M,N,C)
  DIMENSION A(L,M),B(M,N),R(L,N),S(M)
  DO 3 I=1,L
  DO 1 J=1,M
  S(J)=A(I,J)
  DO 3 J=1,N
  SUM=0.0E0
  DO 2 K=1,M

```

2 SUM=SUM+S(K)*B(K,J)
3 R(I,J)=SUM*C
RETURN
END

MPC 370
MPC 380
MPC 390
MPC 400

MPRT 370
MPRT 380
MPRT 390
MPRT 400
MPRT 410
MPRT 420
MPRT 430

GO TO 1
FORMAT('1',132A1)
FORMAT(9X,'*',/,4X,I3,2X,'*',1P11E11.2)
FORMAT('1PAGE',I2,' RIGHT - - -')
FORMAT(/,5X,'COLUMN',11(4X,I3,4X))
FORMAT(/,3X,'ROW',3X,123('*))
END

3
4
5
6
7

BIBLIOGRAPHY

- [1] Digital Equipment Corporation, PDP8/E, PDP8/M, PDP8/F Small Computer Handbook. Maynard, Massachusetts, 1973.
- [2] C. R. Paul, "Efficient Numerical Computation of the Frequency Response of Cables Illuminated by an Electromagnetic Field", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-22, pp. 454-457, April 1974.
- [3] W. B. Boast, Vector Fields. New York: Harper and Row, 1964.
- [4] J. C. Clements, Computation of the Capacitance Matrix for Dielectric-Coated Wires. MSEE Thesis, University of Kentucky, Lexington, Kentucky, December 1973.
- [5] W. H. Hayt, Engineering Electromagnetics. New York: McGraw-Hill, 1967.
- [6] E. C. Jordan and K. G. Balmain, Electromagnetic Waves and Radiating Systems. Englewood Cliffs, N.J.: Prentice-Hall, 1967.
- [7] A. T. Adams, Electromagnetics for Engineers. New York: Ronald Press, 1971.
- [8] A. T. Adams and J. R. Mautz, "Computer Solutions of Electrostatic Problems by Matrix Inversion", Proceedings 1969 National Electronics Conference, pp. 198-201.

- [9] J. C. Clements, C. R. Paul and A. T. Adams, "Computation of the Capacitance Matrix for Systems of Dielectric-Coated Cylindrical Conductors", IEEE Transactions on Electromagnetic Compatibility. Vol. EMC-17. pp. 238-248, November 1975.
- [10] R. F. Harrington, Field Computation by Moment Methods. New York: Macmillan, 1968.
- [11] R. M. Fano, L. J. Chu and R. B. Adler, Electromagnetic Fields, Energy and Forces. New York: Wiley, 1960.
- [12] B. Noble, Applied Linear Algebra. New Jersey: Prentice-Hall, 1969.
- [13] C. R. Paul, "Useful Matrix Chain Parameter Identities for the Analysis of Multiconductor Transmission Lines", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-23, No. 9, pp. 756-760, September 1975.
- [14] W. E. Cramer, Inversion of Block Toeplitz Matrices for Electrostatic Problems. MSEE Thesis, Syracuse University, May 1973.
- [15] W. T. Weeks, "Multiconductor Transmission Line Theory in the TEM Approximation", IBM Journal of Research and Development, pp. 604-611, November 1972.
- [16] International Business Machines, System/360 Scientific Subroutine Package, Version III, Programmer's Manual. White Plains, N.Y., 1968.
- [17] D. H. Sinnott, "Matrix Analysis of Linear Antenna Arrays of Equally Spaced Elements", IEEE Transactions

on Antennas and Propagation, Vol. AP-21, No. 3, pp.
385-386, May 1973.

- [18] S. Ligh, "Centro-Symmetric Matrices", Undergraduate Mathematics Journal, Vol. 3, No. 4, pp. 33-37, Fall 1973.
- [19] C. R. Paul, Applications of Multiconductor Transmission Line Theory to the Prediction of Cable Coupling. Vol. I, Multiconductor Transmission Line Theory, Technical Report, Rome Air Development Center, Griffis AFB, N.Y., to appear.

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